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OPTIMAL ESTIMATION DESIGN OF A CLASS OF SYSTEMS WITH NOISE COUPLING INPUT

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ABSTRACT: An investigation about the robust estimation of a class of systems with noise coupling input saturation is presented in this study. In general, the existed estimation algorithm is based on the exactly known of the input saturation, but in fact, this is not always true in some practical cases because of the coupling of input saturations and input noises. For treating these kinds of coupling problems in the state estimations of systems, in this study, one fuzzy-based optimal estimation algorithm is proposed. The proposed optimal estimator includes two parts: firstly, a regression form fuzzy system that is adopted to approximate the unknown input saturation, and then, an optimal estimator that combines the above fuzzy system by H_2 filter design concept for precisely estimating system states is proposed. This combination of fuzzy approach and H_2 filtering technologies successfully offers one a more simple and practical method for treating the estimation problem of a class of systems that input saturations and input noises couple together.

Keyword: optimal Estimation, Fuzzy Approach, Signal coupling, Filter, Fuzzy approximation, H_2

INTRODUCTION

This paper mainly focuses on the optimal estimator design of a class of systems with coupling of the input noise and input saturation described by

$$X(k+1) = FX(k) + G(sat(u(k)) + w(k)) \qquad (1)$$

where $X(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the input signal, and $w(k) \in R^P$ is the input noise. The saturation function sat of the input signal $R^m \to R^m$ is defined as

$$sat(u(k)) = \begin{bmatrix} sat(u_1(k)) & sat(u_2(k)) & \dots & sat(u_m(k)) \end{bmatrix}^T$$

with $sat(u_i(k)) = \mathrm{sgn}(u_i(k)) \min(\rho, |u_i(k)|)$.

There are, in practice, so many control systems possessing this kind of special feature, e.g., ballistic missile's maneuver couples with wind gusts, acceleration signal measured by accelerometers couples with the external and internal noises, and so on. Generally, the input signal $u(k)$ is always assumed as an exactly known variable and never corrupted with noise; hence one is capable of dealing with these kinds of estimation problems by the well-known Kalman Filter that is widely used in the state estimation. Of course, it is no doubt that in the presence of unknown noise coupling input saturations, performance of Kalman Filter will be seriously degraded since the unknown input saturations coupling with input noises appear on a system model as extensive noises, and the constant processing noise variance will be not capable of covering it because of the time-variant character of these type signals.

Based on the reasons depicted above, it is highly desirable to apply advanced estimation techniques to develop an effective estimator to improve the observation performance for this coupling problem of noises and input saturations. Firstly, how to mimic the unknown input saturation is clearly one of the main tasks for this state estimation problem, since it tremendously affects the performance of a designed estimator. For solving this system modeling problem, the fuzzy approximator is adopted naturally. Recently, fuzzy modeling has been proven as a powerful and useful universal approximator [1-2] for nonlinear systems and any nonlinear system can be approximated by this modeling technology as precise as possible. In this investigation, the unknown input saturation $sat(u(k))$ will be regarded as a black-box and modeled by one multi-input and multi-output (MIMO) fuzzy system. Based on this setting, the estimation problems of a class of systems with the noise coupling input saturations can then be reformulated as fuzzy modeling and noise elimination problems.

PROBLEM FORMULATION

Consider a discrete-time system in the presence of input noise $w \in R^P$, measurement noise $v \in R^l$, and unknown saturated deterministic input signal $sat(u(k)) \in R^m$

$$X(k+1) = FX(k) + G(sat(u(k)) + w(k)) \quad (2a)$$

$$Z(k) = HX(k) + v(k) \quad (2b)$$

$$S(k) = DX(k) \quad (2c)$$

where $X(k) \in R^n$ is the state vector, $Z(k) \in R^n$ is measurement output, and $S(k) \in R^k$ is the signal to be constructed by digital estimators. The saturation function sat of the input command $R^m \to R^m$ is defined as

$$sat(u(k)) = \left[sat(u_1(k)) \quad sat(u_2(k)) \quad \ldots \quad sat(u_m(k))\right]^T$$

with $sat(u_i(k)) = \mathrm{sgn}(u_i(k))\min(\rho, |u_i(k)|)$, and the illustration of the coupling signal $sat(u(k)) + w(k)$ of the input saturation and input noise is shown as Fig. 1. D is a constant matrix, which is specified to extract the desired signal $S(k)$ from the state vector $X(k)$. The process noise $w(k)$ and the measurement noise $v(k)$ are modeled to mutually independent and white Gaussian: $w(k) \sim \delta(0,q)$, $v(k) \sim \delta(0,r)$. When the coupling signal of input saturation and input noise $\{sat(u(k)) + w(k)\}$ occurs in (2), the standard Kalman Filter can not reconstruct the original processing because of the time-varying noise variance $q(k)$.

For treating this time-varying noise variance and signal coupling problem from an optimal design angle, we deal with it firstly by mimicking the behavior of the unknown deterministic input saturation $sat(u(k))$ with a MIMO fuzzy model. Suppose a multi-input and multi-output (MIMO) fuzzy model $\tilde{u}(k,\Theta)$ with respect to the real unknown $sat(u(k))$ exists, hence (2a) can be rewritten as

$$X(k+1) = FX(k) + G(\tilde{u}(k,\Theta) + u_\Delta(k) + w(k)) \quad (3)$$

where $\tilde{u}(k,\Theta) = \Theta^T \varsigma(k)$ is the MIMO fuzzy-based input saturation to mimic the real input saturation $sat(u(k))$ and $u_\Delta(k) = sat(u(k)) - \Theta^T \varsigma(k)$ is the modeling uncertainty between the unknown deterministic input saturation and the fuzzy modeling system.

The time-varying approximated input saturation $\tilde{u}(k,\Theta)$ for the design of the fuzzy-based robust estimator is inferred by a MIMO fuzzy system, for which the *jth* fuzzy IF-THEN rule is represented by

R_j: If $\Delta_1(k)$ is A_{1j} and $\Delta_2(k)$ is $A_{2j} \cdots$ and $\Delta_g(k)$ is A_{gj}, Then $\tilde{u}$ is u_j

where the premise variables $\Delta_g(k) = X_i(k) - \hat{X}_i(k-1)$, $\tilde{u}$ is input signal. A_{ij}, $i \in \{1,\ldots,g\}$ and $j \in \{1,2,\ldots,M\}$, are fuzzy sets, and through this paper, any one of them has the Gaussian membership function with center C_{ij} and standard deviation σ_{ij} as follows :

$$\theta_{A_{ij}}(\Delta_i(k)) = \exp\left[-\tfrac{1}{2}\left(\frac{\Delta_i(k) - C_{ij}}{\sigma_{ij}}\right)^2\right] \quad (4)$$

The fuzzy-based input saturation $\tilde{u}_i$ with center-average defuzzifier, product inference and singleton fuzzifier are given in the following form:

$$\tilde{u}_i(k) = \frac{\sum_{j=1}^{M} u_j \left\{\{\prod_{i=1}^{g} \theta_{A_{ij}}(\Delta_i(k))\}\right\}}{\sum_{j=1}^{M}\{\prod_{i=1}^{g} \theta_{A_{ij}}(\Delta_i(k))\}} \quad (5)$$

Let us denote the fuzzy basis functions as

$$\xi_j(k) = \frac{\prod_{i=1}^{g} \theta_{A_{ij}}(\Delta_i(k))}{\sum_{j=1}^{M}\{\prod_{i=1}^{g} \theta_{A_{ij}}(\Delta_i(k))\}},$$

for $j = 1, 2, \ldots, M$ (6)

and denote.

$$\xi(k) \triangleq [\xi_1(k)\ \xi_2(k) \dots \xi_M(k)]$$

Consequently, $\tilde{u}_i(k)$ in (5) is of the following form:

$$\tilde{u}_i(k) = [\xi_1(k)\ \xi_2(k) \dots \xi_M(k)] \times \begin{bmatrix} u_{i1} \\ u_{i2} \\ \cdot \\ \cdot \\ \cdot \\ u_{iM} \end{bmatrix}$$

$$\triangleq \xi(k)\theta_i$$

where $\theta_i = [u_{i1} \quad u_{i2} \quad . \quad . \quad . \quad u_{iM}]^T$.

Therefore, the fuzzy command $\tilde{u}$ is as the following:

$$\tilde{u}(k,\Theta) = \begin{bmatrix} \tilde{u}_1(k) \\ \vdots \\ \tilde{u}_m(k) \end{bmatrix} = \begin{bmatrix} \xi(k)\theta_1 \\ \vdots \\ \xi(k)\theta_m \end{bmatrix} \tag{7}$$

i.e.,

$$\tilde{u}(k,\Theta) = \Theta^T \varsigma . \tag{8}$$

where

$$\varsigma = \begin{bmatrix} \xi(k) & 0 & \cdots & 0 \\ 0 & \xi(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi(k) \end{bmatrix}, \Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{bmatrix}$$

Define the estimation error as follows:

$$\begin{aligned} e(k) &= S(k) - \hat{S}(k) \\ &= DX(k) - D\hat{X}(k) \end{aligned} \tag{9}$$

where $\hat{S}(k)$ is the reconstructive signal.

(i) Optimal Estimation Problem of a class of systems with noise coupling input saturation

After the above arrangements about the unknown input saturation by the so-called fuzzy modeling technique and the definition of estimation error, then according the H_2 estimation design concept, a robust estimator can then be constructed. The design objective of this study is to find an estimator satisfy the following H_2 performance for all $w(k)$ and $v(k)$.

$$\min_{w(k),v(k)} e(0)^T P_0 e(0) + \sum_{k=1}^{kf} e(k)^T Q e(k) \tag{10}$$

where P_0 is an initial weighting matrix that is assumed to be symmetric and positive definite, and Q is a symmetric semi-positive definite matrix.

(ii) Optimal Estimator Design

The following fuzzy-based robust estimator is proposed to deal with the state estimation of the system in (2).

$$\begin{aligned} \hat{X}(k+1) &= F\hat{X}(k) + L(Z(k) - \hat{Z}(k)) + \tilde{u}(k,\Theta) \\ \hat{S}(k) &= D\hat{X}(k) \end{aligned} \tag{11}$$

where L is the designed robust estimation gain.

Then, it follows from (2), (9) and (11) that

$$\begin{aligned} e(k+1) &= Fe(k) + LH(e(k) + v(k)) + u_\Delta(k) + Gw(k) \\ &= \bar{A}e(k) + u_\Delta(k) + \Omega\bar{w}(k) \end{aligned} \tag{12}$$

where

$$\bar{A} = [F + LH],\ \Omega = [G \quad LH], \bar{w}(k) = [w(k) \quad v(k)]^T$$

Besides, stability is always an important issue in this noise coupling input saturation estimation design problem. In the following, we propose specifying the steady state estimation gain L to stabilize the error dynamic system in (12) with the guarantee of H_2 performance index (10) or (11).

In this section, we will use the common Lyapunov function to check the stability of the system (12). Let P be a positive definite matrix and define the Lyapunov function candidate as follows:

$$V(k, e(k)) = e(k)^T Pe(k) \tag{13}$$

Denote

$$\Delta V(k, e(k)) = V(k+1, e(k+1)) - V(k, e(k)) \tag{14}$$

After some mathematical manipulations, we obtain the following theorem.

Theorem 1: If the fuzzy-based optimal estimator (11) is proposed for a class of systems that input saturations and input noises are coupled naturally, and there exists a positive-definite matrix $P = P^T > 0$ such that the following matrix inequality:

$$\begin{bmatrix} P - DQD + \alpha^2 & 0 & 0 & F^T P - G^T Y^T \\ 0 & I & 0 & BP \\ 0 & 0 & I & -Y \\ PF - YG & PB & -Y & P \end{bmatrix} > 0 \qquad (15)$$

is satisfied, where $Y = PL$, and robust estimation gain $L = P^{-1}Y$. Then the error dynamic system in (13) quadratically stable in the absence of noises and modeling uncertainties, and optimal performance index (10) or (11) is achieved.

Based on the analysis above, the optimal estimation design of a class system with noise coupling input saturation can be summarized as follows:

Step 1) Select the fuzzy rules and membership functions for the input saturation $sat(u(k))$. To reduce the design effort and complexity, rules of the fuzzy modeling system $\tilde{u}(k,\Theta)$ are generally used as few as possible. In general, one can easily obtain a set of rules after some tests by ANFIS algorithm [2].

Step 2) Select the weighting matrices Q.

Step 3) Solve the EVP in (16) to get positive-definite matrix P and Y.

Step 4) Obtain the robust estimation parameter $L = P^{-1}Y$ and corresponding minimum ρ^2.

Step 5) Realize the fuzzy-based robust optimization estimator

$$\hat{X}(k+1) = F\hat{X}(k) + L(Z(k) - \hat{Z}(k)) + \tilde{u}(k,\Theta) \qquad (16)$$

SIMULATION RESULTS

In this research, an estimation problem of a maneuvering target with noise coupling input saturations (maneuvering commands) is given for verifying the performance of the designed method.

To demonstrate estimation performance of the proposed fuzzy-based optimal estimation design, the following scenario is considered.

Case 1: The maneuvering target is toward to aim ($a_r(k) < 0$)

$$r(\mathbf{k}) = 10 \text{ km} \ \theta(k) = 60^\circ \ \ \dot{r}(k) = -1 \text{ km/s}$$

and the initial conditions of $\hat{X}(k)$ and sampling rate T are

$$\hat{r}(k) = 12 \text{ km} \ \hat{\theta}(k) = 55^\circ \ \hat{\dot{r}}(k) = -1 \text{ km/s}, \ T = 0.01 \text{sec}$$

From the simulation results with respect to the range and bearing of a maneuvering target shown in Fig. 2 and 3, the proposed method reveals the precisely estimating property for the target states.

CONCLUSIONS

A novel and robust estimator with a fuzzy approximator for the state estimation of a class of systems with unknown noise coupling input saturations is investigated in this study. From the simulation results of the trajectory tracings of a maneuvering target, this fuzzy-based optimal estimator yields high accuracy estimation with respect to effects of unknown input saturations, input noises and measurement noises. It can thus be claimed that the proposed method possesses strong

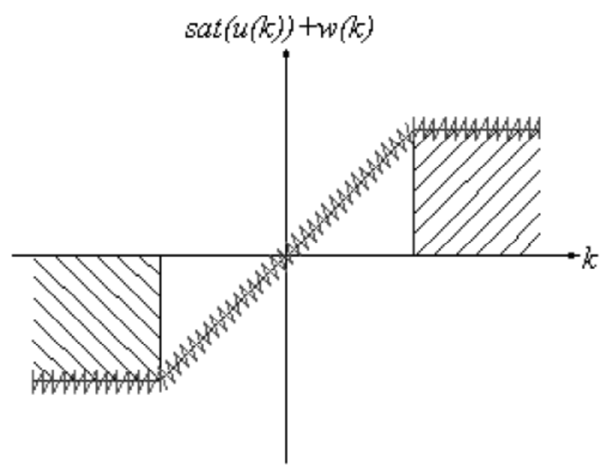

Figure 1: The Nonlinearity Model of ($sat(u(k)) + w(k)$). (Coupling signal of input saturation and noise)

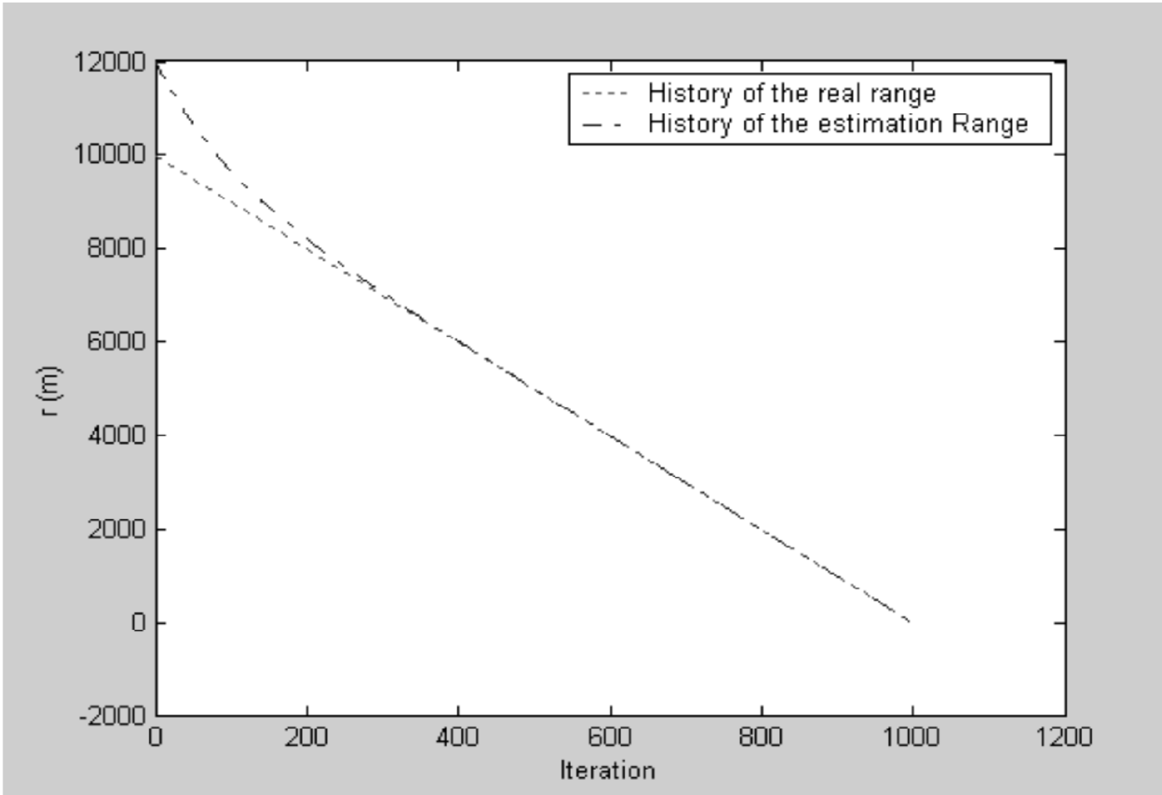

Figure 2: History of the Real Range $r(k)$ and Estimation range $\hat{r}(k)$.

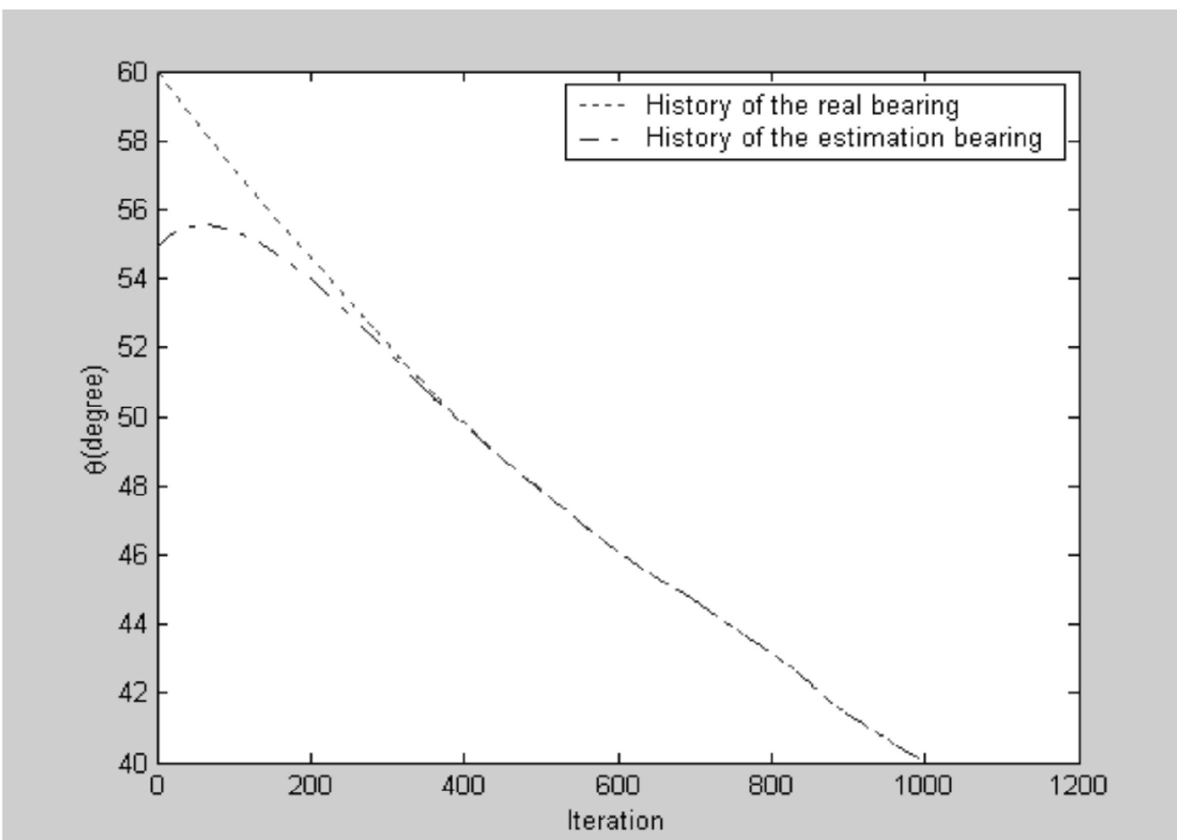

Figure 3: History of the Real Bearing $\theta(k)$ and Estimation bearing $\hat{\theta}(k)$.

potential to be applied in the high-performance estimation design of a class of system with noise coupling input saturations.

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