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# A NOVEL FUZZY CONTROL LAW FOR NONHOLONOMIC MOBILE ROBOTS

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***ABSTRACT:*** A novel fuzzy based control law for the trajectory tracking design of nonholonomic mobile robots is presented in this paper. This approach can be applied to generate trajectory tracking control commands on nonholonomic mobile robot movement. The design objective is to specify one fuzzy control law that can force a mobile robot to follow a predefined trajectory for the nonlinear trajectory tracking control of nonholonomic mobile robot. In general, it is hard to obtain the closed-form solution from this nonlinear trajectory-tracking problem; hence, we try to treat this trajectory tracking problem from the so-called fuzzy control design concepts. Finally, one testing condition: circular reference trajectory is used for performance verifications.

***Keywords:*** nonholonomic mobile robot, fuzzy control law, trajectory following

## 1. INTRODUCTION

In the past decades, wheeled mobile robots were widely applied in various industrial and service fields which include transportation, inspection and security etc, and attract a lot of attentions. Hence, it becomes more and more important at accurate manipulations of wheeled mobile robots, especially in the trajectory tracking subject. Many existing studies [1-7] about trajectory tracking problem are discussed, but in general, they are too complicated and not easily implemented. Based on this reason, we try to propose a novel fuzzy control law to treat the trajectory tracking problem of nonholonomic mobile robots.

This paper will be organized as the following sections: the mathematical model and design objective of nonholonomic mobile robot will be briefly introduced in section II. Fuzzy controller design for predefined trajectory tracking will be described in section III, and simulation results of the nonholonomic mobile robot by the proposed design are demonstrated in section IV. Finally, the conclusions are summarized in section V.

## 2. MATHEMATICAL MODEL AND DESIGN OBJECTIVE

### 2.1. Model and Dynamics of a Nonholonomic Mobile Robot

In general, the structure of wheeled mobile robot consists of two driving wheels which locate at the same axis and a passive self-adjusted supporting wheel which leads the mechanical system. Both driving wheels which are for the motion and orientation purpose are driven by two actuators (e.g. DC motors) independently.

As Fig. 1 shows the two driving wheels with the same radius denoted by r and separated by 2R. The location of the vehicle in the global coordinate frame $\{O, X, Y\}$ is represented by the vector $P = [x_c \quad y_c \quad \theta]^T$, where $x_c$, $y_c$ are the coordinates of the point $C$ in the global coordinate frame and θ is the orientation of the local frame $\{C, X_C, Y_C\}$. The generalized coordinate of the vehicle is described as

$$q = \begin{bmatrix} x_c & y_c & \theta \end{bmatrix}^T \tag{2.1}$$

For the ordinary mobile robot system, the robot just can move as the direction of the axis of the driving wheels with pure rolling and nonslipping nonholonomic condition status.

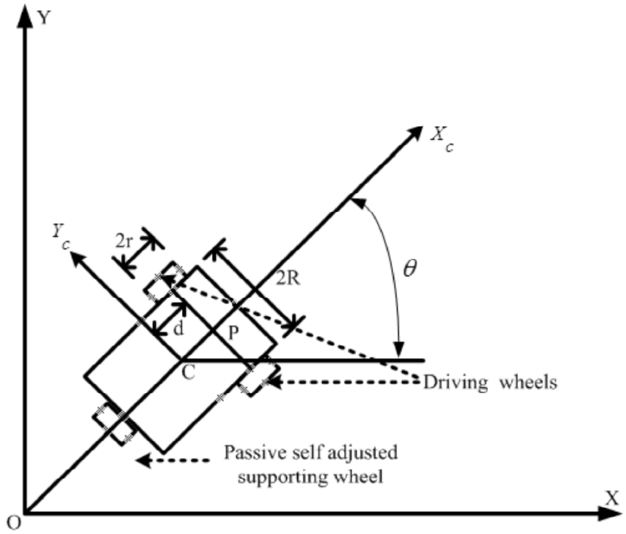


**Figure 1:** Nonholonomic Mobile Robot

Consequently, the velocity of contact point with the ground and orthogonal to the plane of the wheel is zero. We can express as following [8]

$$\dot{y}_c \cos\theta - \dot{x}_c \sin\theta - d\dot{\theta} = 0 \tag{2.2}$$

and then the kinematic equation can also be described as

$$\dot{q} = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_l \\ w \end{bmatrix} \tag{2.3}$$

where $v_l$ and $w$ are the linear and angular velocity along the robot axis.

In this paper, we develop a fuzzy technique to solve this kind of problem, so the mobile robot dynamic equation can be described as [9]

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} = B(q)\tau \tag{2.4}$$

where $H(q) \in \Re^{nxn}$ is a symmetric positive define inertia matrix, $C(q,\dot{q}) \in \Re^{nxn}$ is the centripetal and coriolis matrix, $B(q) \in \Re^{nxn}$ is the input transformation matrix, and $\tau \in \Re^{rx1}$ is the input vector, where $\tau_r$ and $\tau_l$ represents right and left wheel torques, respectively.

$$H(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & 0 & -md\cos\theta \\ md\sin\theta & -md\cos\theta & I \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & md\dot{\theta}\cos\theta \\ 0 & 0 & md\dot{\theta}\sin\theta \\ 0 & 0 & 0 \end{bmatrix}$$

$$B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \sin\theta \\ R & -R \end{bmatrix} \tag{2.5}$$

### 2.2. Problem Formulation

We will develop a fuzzy trajectory tracking control design in this paper. The desired tracking reference trajectory $q_r$ is supposed to be existed into bounded time functions of position vector $q_r \in C^2$ which is a twice continuously differentiable function. The velocity vector and acceleration vector of $q_r$ can be expressed as $\dot{q}_r$ and $\ddot{q}_r$ respectively.

As following is the tracking error definition:

$$e = \begin{bmatrix} \dot{\tilde{q}} \\ \tilde{q} \end{bmatrix} = \begin{bmatrix} \dot{q} - \dot{q}_r \\ q - q_r \end{bmatrix} \tag{2.6}$$

and tracking error dynamic equation is given as

$$\dot{e} = \begin{bmatrix} -H^{-1}(q)C(q,\dot{q}) & 0_{2\times2} \\ I_{2\times2} & 0_{2\times2} \end{bmatrix} e + \begin{bmatrix} -\ddot{q}_r - H^{-1}(q)C(q,\dot{q})\dot{q}_r \\ 0_{2\times2} \end{bmatrix} + \begin{bmatrix} H^{-1}(q)B(q)\tau \\ 0_{2\times2} \end{bmatrix} \tag{2.7}$$

The design objective is then to design a fuzzy based control law to for the tracking error in (2.6) to zero and let the mobile robot successfully follows the desired trajectory.

## 3. CONTROLLER DESIGN FOR MOBILE ROBOT

### 3.1. Fuzzy Logic Based

*1. Fuzzification*

The input and output variables of a fuzzy system are the linguistic variables because they take linguistic values. The input linguistic variables of fuzzy logic-based include: 1. rotation tracking error angle $\theta$, 2. tracking error in x axis $e_x$ and 3. tracking error in y axis $e_y$; besides the output variable is control torques $\tau \in \Re^{rx1}$. The universe of discourse of the linguistic variable $\theta$, is supposed to be [-20,20] deg, $e_x$ is [-20,20] meter, and $e_y$ is [-20,20] meter. About control output, the universe of discourse is set up as [-10,10]degree for $q$, [-10,10]degree for $e_x$, and [-10,10]degree for $e_y$. All figures are illustrated as the following figures.

The linguistic values taken by these variables are expressed by linguistic sets. Each of linguistic variables is assumed to take five linguistic sets as the above and are defined as large negative (LN), large position (LP), small negative (SN), small position (SP),and zero (ZE).

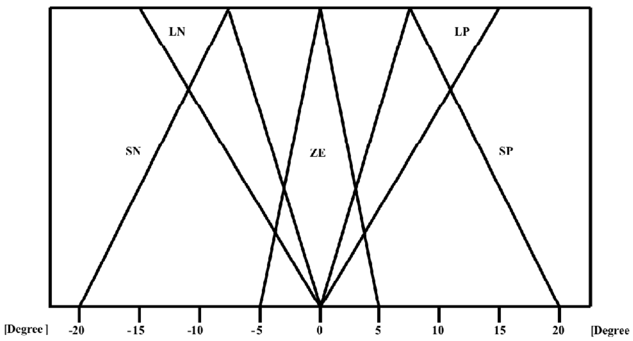


**Figure 2:** Rotation Tracking Error Angle of Mobile Robot

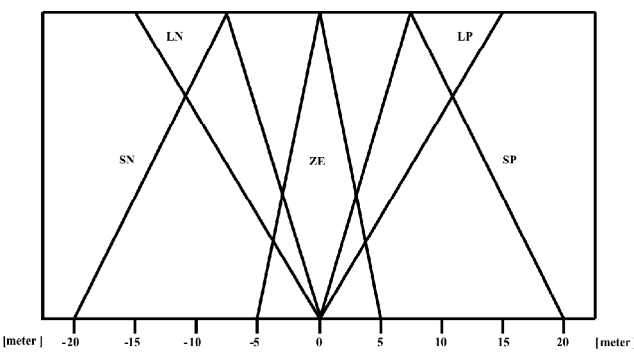


**Figure 3:** Tracking Error in x Axis of Mobile Robot

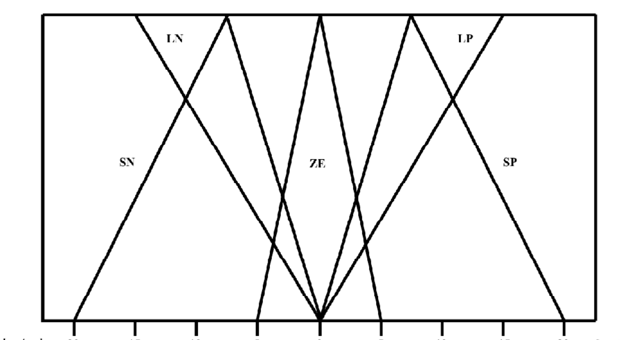


**Figure 4:** Tracking Error in y Axis of Mobile Robot

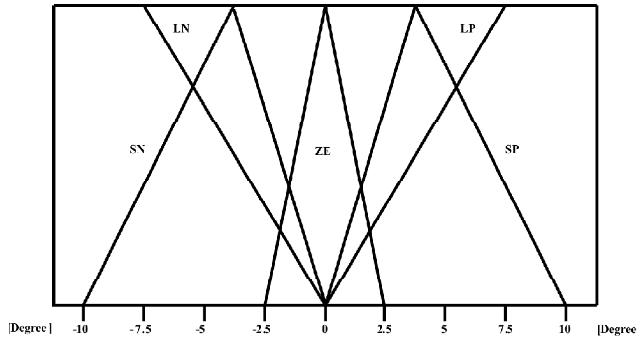


**Figure 5:** The Control Angle Output of Mobile Robot

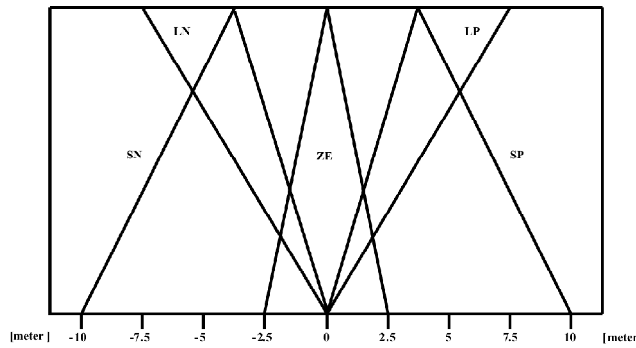


**Figure 6:** The Control x Axis Output of Mobile Robot

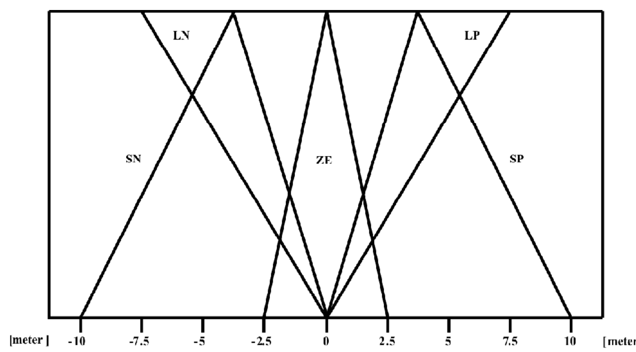


**Figure 7:** The Control y Axis Output of Mobile Robot

The linguistic sets are described by their membership functions as shown in Figs. 2-4. To simplify the computation in the actual operation, triangular membership functions are suggested. It has been found that using complex forms of membership functions, such as bell-shaped functions, cannot bring any advantage over the triangular ones.

## 2. *Defuzzification*

The outputs of the linguistic rules are fuzzy, but the control command must be crisp. Therefore, the outputs of the linguistic rules must be defuzzified before feeding the mobile robot plant. The crisp control action is calculated here using the c.g.center-of-area (COA) defuzzification procedure.

## 4. SIMULATION RESULTS

In this section, the proposed control law will be verified by tracking a predefined circle trajectory as Fig. 8.

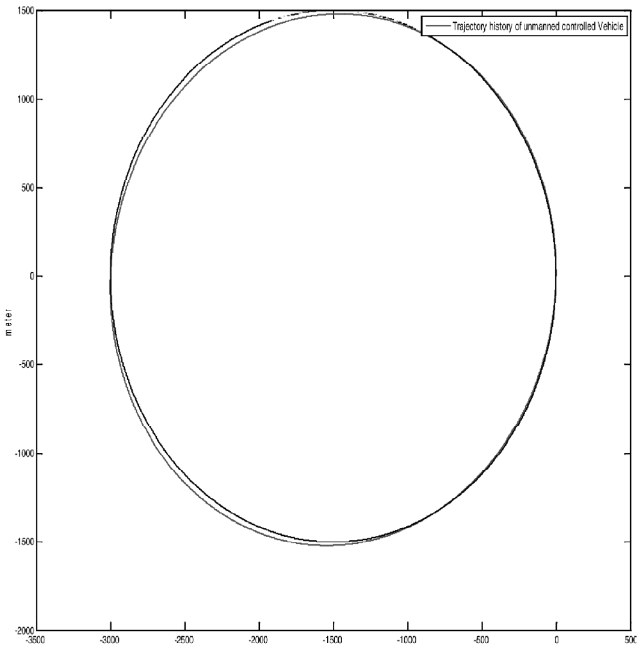


**Figure 8:** Predefined Trajectory (red-line) and Tracking Result (Black-line)

In this scenario, the controlled mobile robot should precisely follow this predefined trajectory under the force of the proposed method. From the simulation result, it is obvious that this proposed method can direct the controlled mobile robot to track the circular trajectory precisely with only a few of energy consumptions in x and y axis.

According to the above simulation results, the proposed control algorithm reveals the capability for tracking a predefined trajectory.

## 5. CONCLUSIONS

A novel fuzzy control law is successfully developed for improving the trajectory tracking ability of the nonlinear nonholonomic mobile robot system in this paper. From the simulation results, it is obvious to find out that the proposed fuzzy method achieves a satisfactory performance for tracking the desired trajectory in circular trajectory, and the structure of the control law is relatively easily to be implemented.

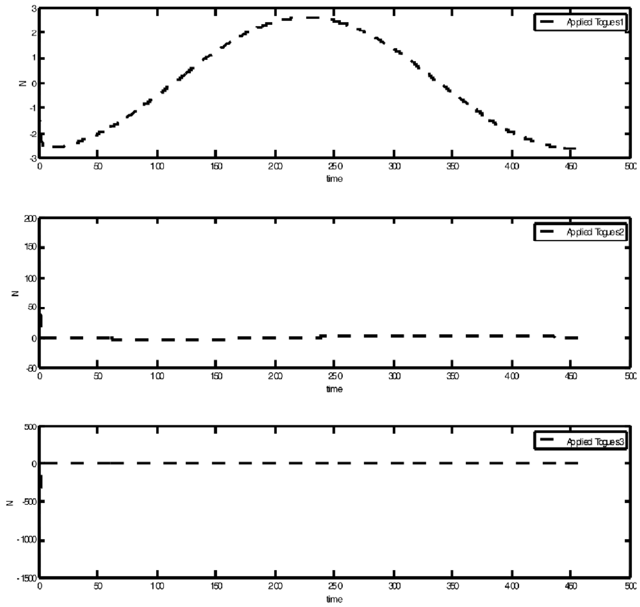


**Figure 9:** Control Histories in Rotation, x axis and y axis


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