International Journal of Electrospun Nanofibers and Applications, Vol. 3, No. 2 (July-December, 2017)
ISSN : 0973-628X

Isologous Fractal Super Fibers or Fractal Super Lattices

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Abstract: *Based on fractal super fibers and fractal super lattices, the concept "isologous fractal set" is abstracted. Through careful inductions and comparisons, we find that identical fractal patterns can be generated from different initial cell elements (hereafter abstracted as cells). In this paper, three classes or six types of cells are involved. They are circular-disk and circular-ring, hexagonal-disk and hexagonal-ring, equilateral triangle-disk and equilateral triangle-ring. Although these cells have divergent shapes and different topologies, they still lead to identical fractal patterns at limit states at where the level numbers of structures approach infinities. The necessary conditions for forming isologous fractal sets are analyzed. The potential applications of isologous fractal sets to nonlinear dynamics are predicted.*

Keywords: *fractal super fibers, fractal super lattices, self-similarity, isologues, fractal sets*

1. INTRODUCTION

An electrospun fiber may have micro voids inside [1, 2]. Usually the distributions of such voids are of fractal-like features and thus the electrospun fiber may form a fractal set. Such fractal electrospun fibers may be regarded as the sound physical background for the idealized fractal fibers [3, 4, 5].

Recently, during the studies on fractal super fibers [4, 5], we reveal the phenomenon of "reaching the same goal by different routes". This phenomenon is interesting but abnormal. "Interesting" means that such phenomenon is seldom reported in classical fractal geometry. "Abnormal" implies that this phenomenon overthrows our commonness: Usually different cells lead to different fractal sets. In this paper, we will try to induce and extend the abnormal phenomenon, and from which we will abstract the concept of isologous fractal sets.

2. THE CONCEPT OF ISOLOGOUS FRACTAL SETS

"Isologue" is a concept both in chemistry and in topology. Its core meaning is as follows: Identical structures may be formed from different cells. This core idea clearly depicts the phenomenon of "reaching the same goal by different routes".

In Refs. [3, 4, 5], several cells have been presented. They may be further extended and classified as follows. According to their topological structures, they may be grouped into two-dimensional disk and one-dimensional ring. According to their geometric shapes, they may be classified into circle, hexagon and equilateral triangle. Thus totally we get three classes or six types of cells: circular-disk and circular-ring (Fig.1), hexagonal-disk and hexagonal-ring (Fig.2), equilateral triangle-disk and equilateral triangle-ring (Fig.3). Obviously, these cells are either different in shapes or distinct in topologies. They are "heteroideus cells". To one's surprise, "heteroideus cells" may lead to "identical fractal sets".

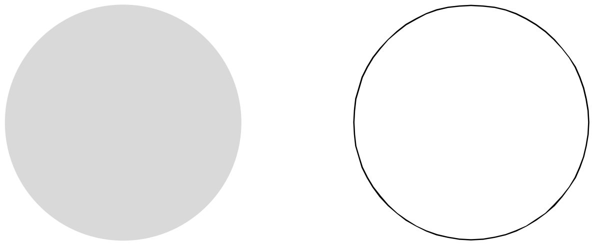

Figure 1: Circular-disk and Circular-ring Cells

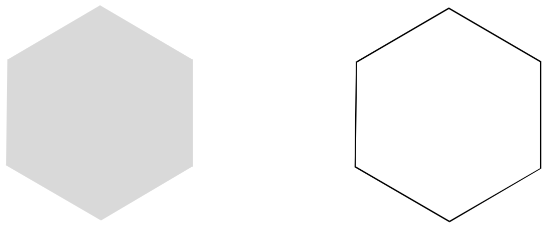

Figure 2: Hexagonal-disk and Hexagonal-ring Cells

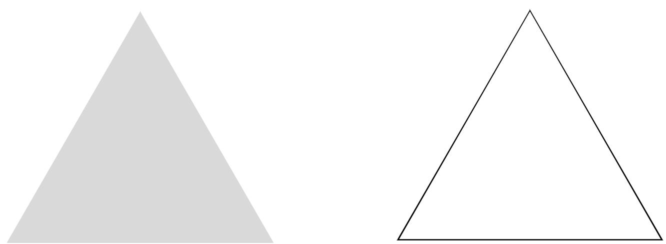

Figure 3: Equilateral triangle-disk and Equilateral Triangle-ring Cells

The six-circle fractal set [4, 5] can be created from circular-disk cell as well as circular-ring cell (Fig. 4, Fig. 5). There are both differences and resemblances in Fig. 4 and Fig. 5. The differences are as follows: (a) The Euclidean dimensions of the cells are different: The circular-disk is two dimensional, while the circular-ring is one dimensional. (b) The topologies of the cells are different: The circular-disk is a plane with boundary, while the circular-ring is a closed curve without boundary. (c) The paths to approach the fractal patterns are distinct: The circular-disk reaches the fractal pattern through planar-disk-set, while the circular-ring approaches the fractal pattern through closed-curve-set. (d) The evolutions for dimensions are different: The circular-disk decreases its dimension to form the fractal pattern, while the circular-ring increases its dimension to form the fractal pattern. (e) The physical correspondences are different. The planar-disk sets can be regarded as the cross-sections of fractal super fibers, while the circular-ring sets can be considered as fractal super lattices [5]. The resemblances of Fig. 4 and Fig. 5 are as follows: (a) The contours of the two cells are identical. (b) The growth modes are the same: Both are inside growths in constrained spaces [5]. (c) The topology evolution modes are the same: Both are anamorphosis evolutions [5]. (d) The self-similar ratios are the same: Both are $r_S = 1/3$. (e) The fractal dimensions are the same: Both are $D_S = 1.6309$. (f) The final fractal pattern are the same [4, 5].

The six-hexagon fractal set can be created from both the hexagonal-disk and the hexagonal-ring (Fig. 6, Fig. 7). The differences and resemblances between Fig. 6 and Fig. 7 are completely correspondent to those between Fig. 4 and Fig. 5, and will not be repeated here.

The six-circle fractal set (Fig. 4, Fig. 5) and the six-hexagon fractal sets (Fig. 6, Fig. 7) have identical fractal patterns. This result is more or less beyond our expectations. Although circular-disk and circular-ring (or the hexagonal-disk and hexagonal-ring) are "heteroideus cells", their differences are not large, and their "isologues" are understandable. However, as "heteroideus cells" the circle and the hexagon have large differences and are difficult to be "isologous". To one's surprise, they still lead to isologous fractal sets. How do we understand this? In fact, the circular sets and the hexagonal sets in Fig. 4-Fig. 7 are of one-to-one correspondences. When the level number reaches infinity, the circles and hexagons will contract to identical points. In another word, the circular sets and hexagonal sets will converge to two identical fractal point sets.

Among various differences between the two classes of heteroideus cells, the planar-disk-approach and closed-curve-approach are very unique. The former reduces its dimension and the latter increases its dimension, but the final fractal pattern are the same. This looks very strange. In classical fractal geometry, a fractal set is created either by planar approach or by curve approach, and few of them can be formed through contrary approaching paths. A conventional fractal set is generated either through descending dimension or through ascending dimension, and seldom can unify two opposite growth modes. Therefore, the isologous fractal sets in this paper are heuristic.

The third class of cells is equilateral triangle-disk and equilateral triangle-ring (Fig. 3). Based on the two cells, the six-triangle fractal sets can be created (Fig. 8 and Fig. 9). The similarities and differences between Fig. 8 and Fig. 9 are also correspondent to those between Fig. 4 and Fig. 5. Besides, Fig. 8 and Fig. 9 are especially distinct in their growth modes and evolution processes. The growth in Fig. 8 can be realized by "one-step operation": If a congruent equilateral triangle-ring is upended on every equilateral triangle-ring, then higher level structure may be created. However, the growth in Fig. 9 needs "two-step operation": In the first step, three congruent equilateral triangles are cut out symmetrically from every equilateral triangle-disk. In the second step, the three

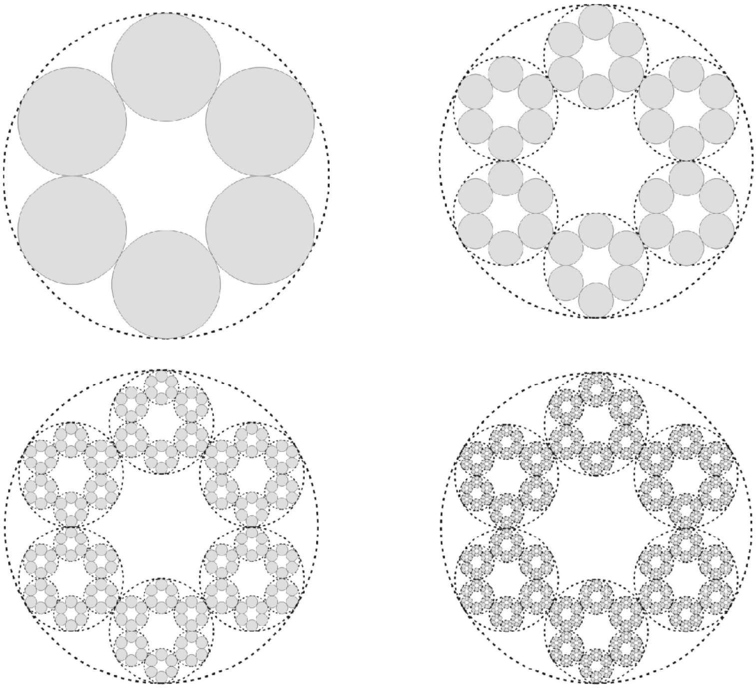

Figure 4: Six-circle Fractal Set Formed from Circular-disk Cell

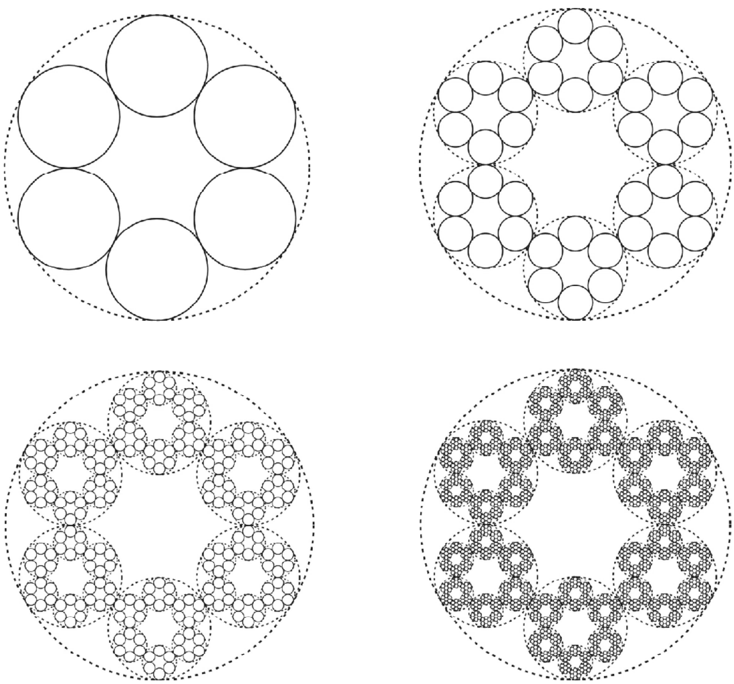

Figure 5: Six-circle Fractal Set Formed from Circular-ring Cell

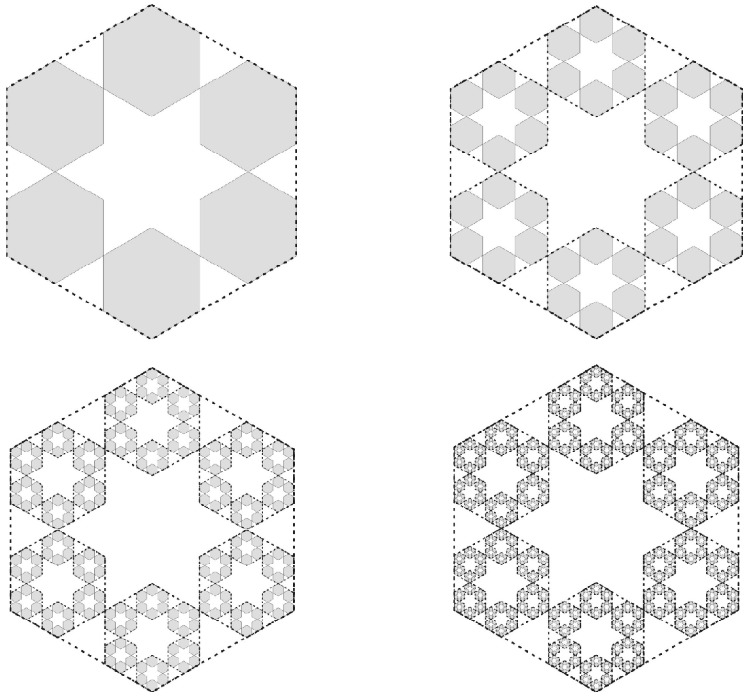

Figure 6: Six-hexagon Fractal Set Formed from Hexagonal-disk Cell

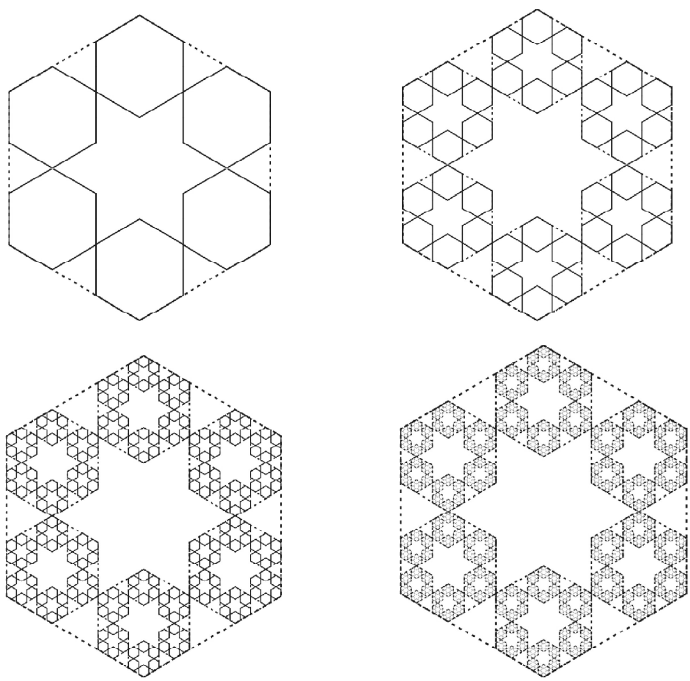

Figure 7: Six-hexagon Fractal Set Formed from Hexagonal-ring Cell

residual equilateral triangles are rotated 180° outwards around their lateral axes respectively. Then higher level structure may be generated.

Among all the figures, Fig. 4-Fig. 8 are the outputs of "one-step operation", and only Fig. 9 is the output of "two-step operation". From the viewpoint of kinematics, the "one-step operation" in Fig. 4-Fig. 8 is an in-plane movement. The "two-step operation" in Fig. 9 is the combination of two different movements: The first operation (i.e. the "cutting out") is an in-plane movement, while the second operation (i.e. the rotation) is a out-plane movement. In fractal geometry, "one-step operation" and in-plane movement are very common, but "two-step operation" and out-plane movement are seldom reported. It should be noted that Fig. 8, Fig. 9 and Fig. 4-Fig. 7 are all "isologous fractal sets". This means that different operations and different kinematics can lead to identical fractal sets, which is very interesting.

Similar to Fig. 5 and Fig. 7, the geometric pattern in Fig. 8 is physically a kind of lattice structure with self-similar symmetry. The first level structure in Fig. 8 is exactly the Kagome lattice that is widely used in modern industries. In Ref. [5], Fig. 8 is called the fractal super Kagome lattice, and from which the concept of fractal super lattice is abstracted [5]. Thus Fig. 5, Fig. 7 and Fig. 8 are all isologous fractal super lattices. A fractal lattice is actually a fractal crystal. However, different from a classical crystal in physics that obeys translational symmetry or rotational symmetry, a fractal crystal follows self-similar symmetry or affine symmetry.

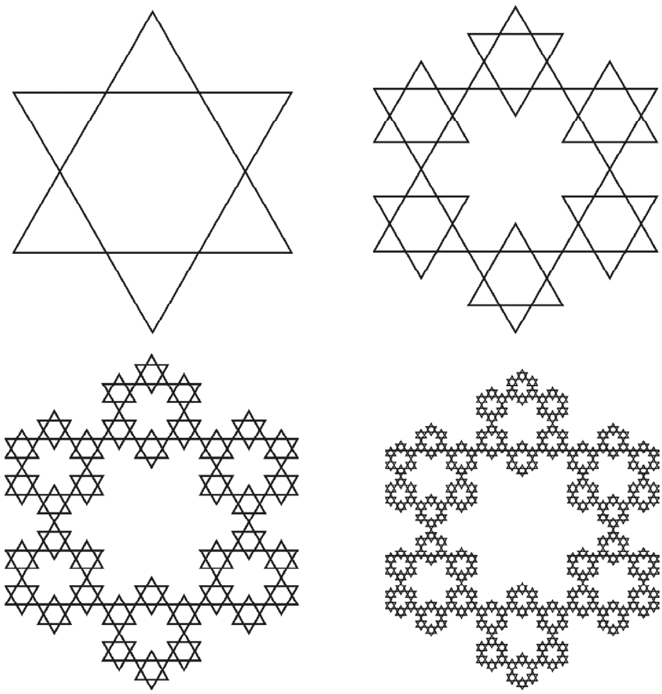

Figure 8: Six-triangle Fractal Set (Fractal Super Kagome Lattice) Formed from Equilateral Triangle-ring Cell

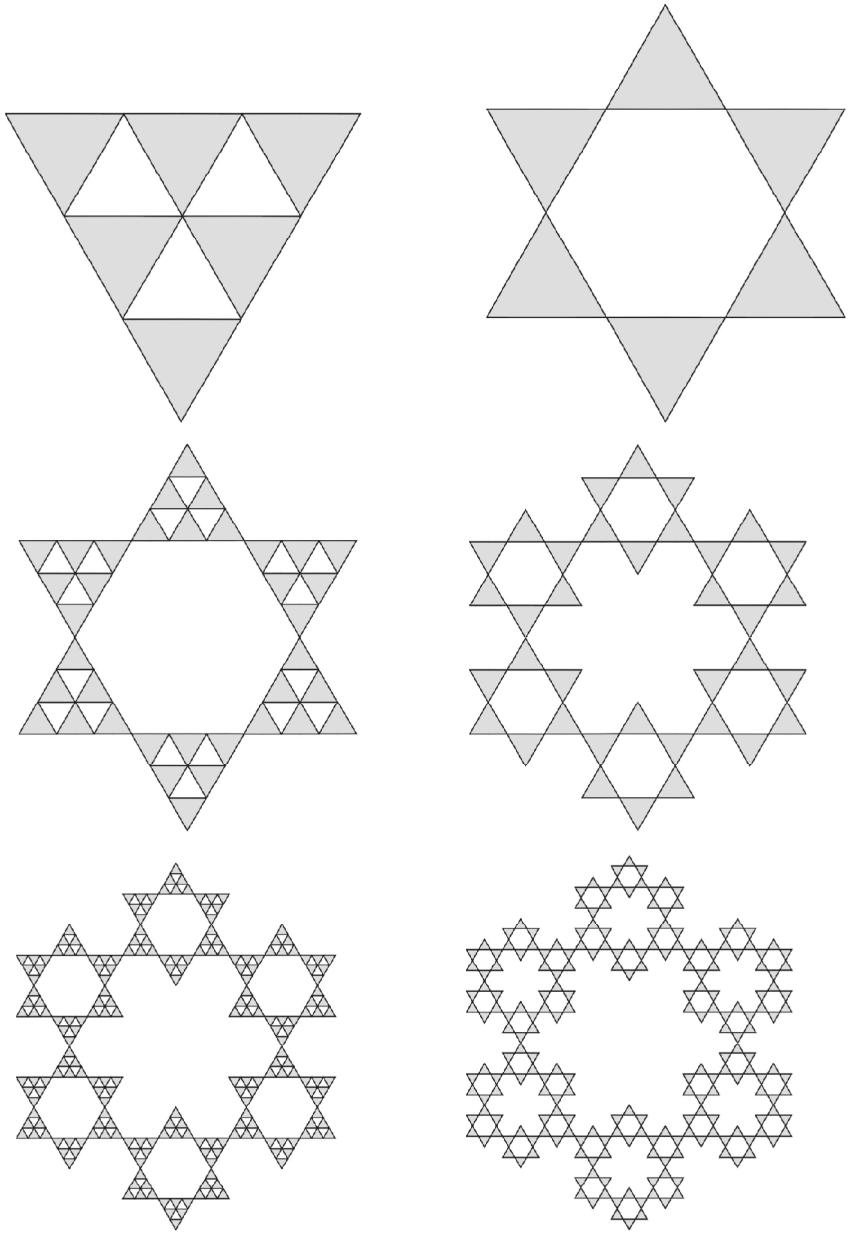

Figure 9: Six-triangle Fractal Set Formed from Equilateral Triangle-disk Cell

3. NECESSARY CONDITIONS AND POTENTIAL APPLICATIONS OF ISOLOGOUS FRACTAL SETS

To generate identical fractal sets with isologues, the geometric sizes of cells have to satisfy the necessary condition —— the hexagonal cell and equilateral triangle cell have to inscribe the circular cell. Suppose the radius of the circular cell is r_0, the length of side of the hexagonal cell is a_{06}, and the length of side of the equilateral triangle cell is a_{03}, then the necessary condition may be written as:

$$a_{06} = r_0 = \frac{\sqrt{3}}{3} a_{03} \tag{1}$$

The isologous fractal sets above have a common point: Their fractal patterns are all distributed inside a planar torus closed by an outer-ring with radius r_{out} and an inner-ring with radius r_{in} (Fig. 10):

$$r_{out} = r_0, \quad r_{in} = \frac{1}{3} r_0 \tag{2}$$

Formulation (2) assures:

$$\frac{r_{in}}{r_{out}} = \frac{1}{3} = r_S \tag{3}$$

The ratio of the radius of inner-ring to the radius of outer-ring is exactly the self-similar ratio. This result is clear for Fig. 4-Fig. 7, but is not so obvious for Fig. 8 and Fig. 9.

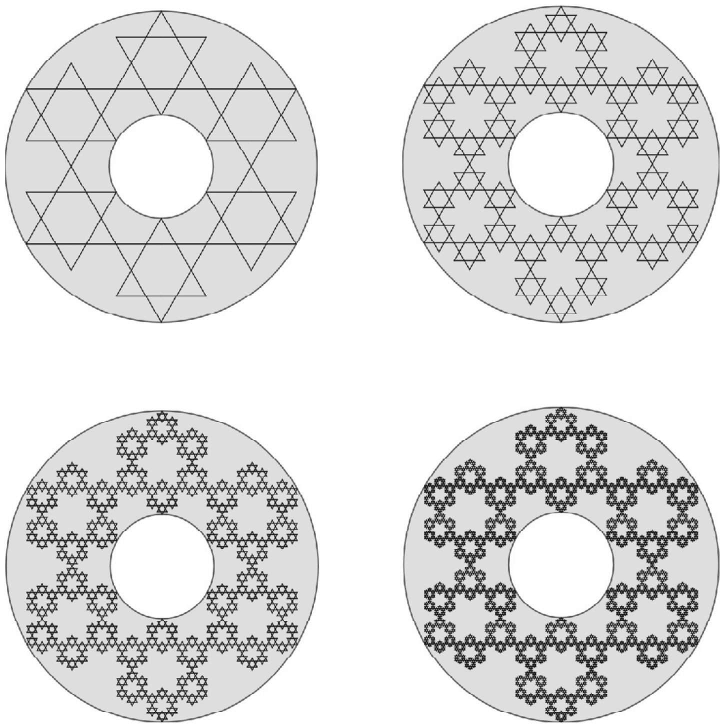

Figure 10: Fractal Pattern Distributed Inside the Planar Torus

An isologous fractal set is similar to a strange attractor. In nonlinear dynamics, although initial disturbances may lead to different dynamic processes, these dynamic processes develop to the same goal finally under the attractions of the strange attractor. It is noted that strange attractors possess fractal structures. Thus isologous fractal sets have potential applications in nonlinear dynamics.

4. CONCLUSIONS

From fractal super fibers and fractal super lattices to isologous fractal sets, a series of new concepts are defined, and a series of novel ideas are inspired. These new concepts and ideas enrich our fractal geometry in some extent. They are possible to become the precursor for industrial applications, bring opportunities for R&D, and provide reference for understanding the complexity of nonlinear dynamics.

ACKNOWLEDGEMENTS

Supports by the Chinese NSFC under Grant No.10572076, No.10872114 and NSF of Jiangsu province under Grant BK2008370 are gratefully acknowledged.

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