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ENTROPY GENERATION FOR MHD RADIATIVE COMPRESSIBLE FLUID FLOW IN A CHANNEL PARTIALLY FILLED WITH POROUS MEDIUM

SHALINI JAIN, VIKAS KUMAR AND SHWETA BOHRA

Abstract: The present study highlights the entropy generation for MHD compressible flow and heat transfer in partially filled channel with porous medium in the presence of radiation. Upper portion is clear fluid region and lower portion is porous region. We analyzed the viscous dissipative effect in the case of shear driven flow: the shear heating generated by the upper plate which is movable along with fluid friction. Analytical solution of the governing equations for momentum and energy has been obtained. The effects of permeability of the porous medium, viscosity ratio parameter, radiation and magnetic field parameter on temperature distribution, heat transfer rate, entropy generation number and Bejan number for both porous and clear fluid regions are obtained and discussed.

1. Introduction

Flow in channels which are partially filled with a porous medium is important because of its many important engineering applications. Flow in channels which are partially filled by a porous medium and partially filled by a clear fluid is important because the occurrence of such fluid flow situations in many engineering applications. Some examples are: drying processes, heterogeneous reactors, filtering, cooling and heating processes, geothermal energy management, chemical reactors, electronic cooling, ceramic processing etc. Recently, in modified equipments the study of porous inserts has acquired substantial interest for improving heat transfer. Beavers and Joseph [1] who has been first analyzed the fluid mechanics at the interface between a fluid layer and a porous medium over a flat plate. Recently, heat transfer in channels partially filled with porous media has received considerable attention and focus of several investigations, Al-Nimr and Alkam [2], Chikh et. al., [3], Vafai and Kim [4], and Poulikakos and Kazmierczak [5]. Coupled and heat transfer problems in channels partially filled by a porous medium were studied analytically by Kim and Russell [6], Chauhan and Gupta [7], Kuznetsov [8] and many others. Use of an external magnetic field is applied in many industrial applications, particularly as a control mechanism in material manufacturing. Magnetic field strength plays an important role for crystal formation. The scientific treatment of the problems formulation of irrigation, tile drainage and soil erosion are the present area for focus of the development of porous channel flow. The MHD channel flow with heat transfer finds applications in thermo fluid transport modeling in meteorology, magnetic geosystems, and solidification process, turbo machinery in metallurgy and in some astrophysical problems. Several researchers McWhirter et. al., [9], Geindreau and Auriault [10],

Keyword. Radiation, compressible fluid, permeability, porous medium, magnetohydrodynamics, entropy generation.

Chauhan and Jain [11], Hayat et. al., [12],Chauhan and Rastogi [13], Marteen [14], Job and Gunakala [15], Onyango et. al., [16], Sreekala and Reddy [17], Kiema et. al., [18], Jain and Choudhary [19] studied two dimensional MHD flow and heat transfer through channels and plates under different boundary conditions.

Heat transfer analysis with viscous dissipative term is important because due to movement of boundary, the velocity of the fluid near the boundary deforms and shears the layer of fluid adjacent to the boundary. Therefore, study of viscous dissipation effects with heat transfer for moving boundaries is significantly important. Brinkman [20] has given first theoretical work on single phase flow and heat transfer study with viscous dissipation effect through a circular tube. Numerical analysis in the presence of viscous dissipation has been given by Cheng and Wu [21] for Newtonian fluid flow in a parallel plate channel. Kundu [22] investigated the effect of viscous dissipation on study of heat transfer between two parallel plates. He considered two cases of fluid flow: shear driven flow and Poiseuille flow. Radiative flows and heat transfer are frequently encountered in many scientific and environmental processes. It plays an instrumental role in designing the pertinent equipment such as heating and cooling of chambers, and solar power technology. Several researchers have examined effects of radiation on study of transfer in porous and non-porous medium by utilizing other radiative flux model or the Rosseland, such as Sparrow and Cess [23], Raptiset al., [24], Chauhan and Rastogi [25], Singh [26], Plumb et. al., [27], Seddeek and Salem [28], Al-Odat et. al., [29], Jain and Bohra [30].

The performance of the engineering processes can be developed by using Second law analysis. It is applicable for investigation the entropy generation rate. Since entropy generation is the measure of the destruction of the system for available work, the determination of the factors responsible for the entropy generation is also useful in upgrading the system performances. The method is introduced by Bejan [31, 32]. The entropy generation is considered in many energy- related applications, such as geothermal energy, solar power collectors and the cooling of modern electronic systems. The irreversibility phenomena, which is expressed by entropy generation in a given system, is related to heat-mass transfers, viscous dissipation and magnetic field etc. Mahmud et. al., [33] has examined thermodynamic analysis of mixed convection in a channel with transverse hydromagnetic effect. In a porous channel the study of entropy minimization for MHD fluid flow has been done by Tasnim et. al., [34]. Makinde and Osalusi [35] analyzed the second law analysis for laminar flow in a channel which is filled with saturated porous media. The parametric analysis of entropy generation rate in a channel has been presented by Cimpean and Pop [36]. The entropy generation rate for radiative MHD Couette flow inside a channel with naturally permeable base has been studied by Vyas and Rai [37]. Several investigators discussed the effects of entropy generation in channels and ducts of different configurations filled with a porous material, such as Al-Odat et. al., [38], Damesh et. al., [39], Das and Jana [40], Rajvanshi et. al., [41], Jain et. al., [42].

In this study, we have examined the entropy generation effect on MHD compressible shear driven fluid flow and heat transfer for moving impermeable wall of a composite channel partially filled with a porous medium and partially with a clear fluid. The influence of radiation and viscous dissipation on the rate of heat transfer has also been investigated. The effects of the permeability parameter, Hartmann number. Brinkman number, radiation parameters are investigated and discussed. The focus of this study is on the radiation & MHD effects on the heat transfer characteristics in a channel through both porous and clear fluid regions. We hope the results of this study may serve as a basis for an experimental test and helpful to engineers in performing preliminary design calculations.

2. Formulation of the Problem and Solution

A steady, laminar, viscous compressible Newtonian fluid flow in an infinitely long impermeable horizontal parallel-plate channel is considered under the effect of transverse magnetic field B_0, applied normal to the flow direction. The physical configuration of the problem is shown in Figure 1. The induced magnetic field is negligible i.e. Hall current effect of MHD is negligible, as magnetic Reynolds number is assumed to be small.

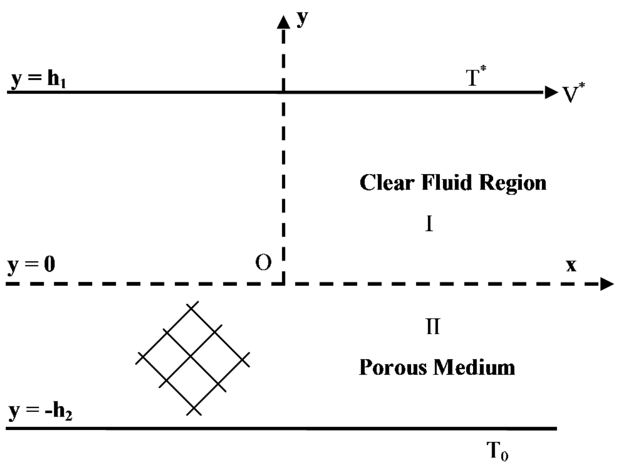

Figure 1. Schematic diagram

The channel is divided into two regions. Region-I ($0 \leq y \leq h_1$) is clear fluid region, where momentum equation is taken as Navier-Stokes equations; and Region-II ($-h_2 \leq y \leq 0$) is porous medium region, where Brinkman equation hold. The upper plate is moving in x-direction with constant velocity V^* and has a uniform temperature T^*, while lower plate is stationary and has a uniform temperature T_0. Here $T_0 < T^*$. The x-axis is taken along the fluid-porous medium interface and y-axis is normal to it. The flow in the channel is assumed to be fully developed and the shear-driven flow caused to the motion of the upper plate along x-direction, thus the only non-vanishing component of the velocity is in the x-direction and both velocity and temperature of the fluid depend only on y. Let u_1, u_2 be the velocity components; t_1, t_2 be the temperatures and P_1, P_2 be the pressure for the clear-fluid region and porous region respectively. We assume that all the thermo-physical properties other than density are constant and behavior of compressible fluid is such that pressure can be defined as a function of density and temperature, i.e., $P_1 = P_1(\rho, t_1)$, $P_2 = P_2(\rho, t_2)$. Also let P_0 be the pressure at (0, 0, 0).

It is further assumed that the medium is optically thick, which is gray, emitting-absorbing radiation but not scattering. Following Rosseland flux model and Seigel and Howell [43], the expression for radiative heat flux takes the form, as follows:

$$q_r = -\frac{4\sigma'}{3k'}\frac{dt^4}{dy} \tag{1}$$

where, σ', k' are Stephan-Boltzman constant and mean absorption coefficient respectively.

We assume that the temperature difference within the fluid is sufficiently small so that t^4 may be expressed as a linear function of temperature t. This can be done by expanding t^4 in Taylor series about T_0 and omitting higher order terms; which yield.

$$t^4 \cong 4T_0^3 t - 3T_0^4 \tag{2}$$

The governing equations for compressible fluid flow; such as, continuity, momentum and energy equations simplify to the following:

The equation of continuity for both regions reduces to

$$\frac{\partial \rho}{\partial x} = 0 \tag{3}$$

For clear-fluid region-I ($0 \le y \le h_1$);

$$\mu\frac{d^2u_1}{dy^2} - \frac{\partial P_1}{\partial x} - \sigma B_0^2 u_1 = 0 \tag{4}$$

$$-\frac{\partial P_1}{\partial y} - \rho g = 0 \tag{5}$$

$$-\frac{\partial P_1}{\partial z} = 0 \tag{6}$$

$$k\frac{d^2t_1}{dy^2} + \mu\left(\frac{du_1}{dy}\right)^2 + \sigma B_0^2 u_1^2 - \frac{dq_r}{dy} = 0 \tag{7}$$

For porous region-II ($-h_2 \le y \le 0$);

$$\bar{\mu}\frac{d^2u_2}{dy^2} - \frac{\mu}{k_0}u_2 - \frac{\partial P_2}{\partial x} - \sigma B_0^2 u_2 = 0 \tag{8}$$

$$-\frac{\partial P_2}{\partial y} - \rho g = 0 \tag{9}$$

$$-\frac{\partial P_2}{\partial z} = 0 \tag{10}$$

$$\bar{k}\frac{d^2t_2}{dy^2} + \frac{\mu u_2^2}{k_0} + \bar{\mu}\left(\frac{du_2}{dy}\right)^2 + \sigma B_0^2 u_2^2 - \frac{d\bar{q}_r}{dy} = 0 \tag{11}$$

The boundary conditions for the present problem are:

at $\quad y = h_1;\ u_1 = V^*,\ t_1 = T^*$

at $\quad y = 0;\ u_1 = u_2,\ \bar{\mu}\dfrac{du_2}{dy} - \mu\dfrac{du_1}{dy} = \dfrac{\beta\mu}{\sqrt{k_0}}u_2,\ t_2 = t_2,\ k\dfrac{dt_1}{dy} = \bar{k}\dfrac{dt_2}{dy},\ P_1 = P_2$

at $\quad y = -h_2;\ u_2 = 0,\ t_2 = T_0 \qquad (12)$

Here μ and $\bar{\mu}$ are the viscosity and effective viscosity, k and $\bar{k}$ are the thermal conductivity and effective thermal conductivity, q_r and $\overline{q_r}$ are radiative heat flux in the clear-fluid and porous region respectively, ρ is the density, k_0 is the permeability of the porous medium, β is the dimensionless empirical constant, and σ is the electric conductivity.

The continuity and above momentum equations imply that

$$\rho = \rho(y),\ P_1 = P_1(\rho(y),\ t_1(y)) \text{ and } P_2 = P_2(\rho(y),\ t_2(y)) \qquad (13)$$

Solving the momentum equations for pressure, we obtain the expressions for the pressure distribution in both regions as follows:

$$P_1(y) = P_0 - g\int_0^y \rho(y)\,dy\ ,\ P_2(y) = P_0 + g\int_y^0 \rho(y)\,dy \qquad (14)$$

Now we introduce the following non-dimensional quantities:

$$u = \frac{u_1}{V^*},\ U = \frac{u_2}{V^*},\ \eta = \frac{y}{h_1},\ a = \frac{h_2}{h_1},\ \phi_1 = \frac{\bar{\mu}}{\mu},\ \phi_2 = \frac{\bar{k}}{k},$$

$$\theta = \frac{t_1 - T_0}{T^* - T_0},\ T = \frac{t_2 - T_0}{T^* - T_0} \qquad (15)$$

Using equations (1), (2), (13) and above non-dimensional quantities, equations (4) - (11) reduces to the following:

$$\frac{d^2u}{d\eta^2} - M^2u = 0 \qquad (16)$$

$$\frac{d^2U}{d\eta^2} - s^2U = 0 \qquad (17)$$

$$\left(1 + \frac{4}{3R}\right)\frac{d^2\theta}{d\eta^2} + N_{Br}\left(\frac{du}{d\eta}\right)^2 + N_{Br}M^2u^2 = 0 \qquad (18)$$

$$\left(\phi_2 + \frac{4}{3R}\right)\frac{d^2T}{d\eta^2} + N_{Br}\phi_1\left(\frac{dU}{d\eta}\right)^2 + N_{Br}\frac{U^2}{K} + N_{Br}M^2U^2 = 0 \qquad (19)$$

and the boundary conditions (12), in non-dimensional form become

at $\quad \eta = 1;\ u = 1,\ \theta = 1$

at $\quad \eta = 0;\ u = U,\ \theta = T,\ \dfrac{dU}{d\eta} - \dfrac{d}{\phi_1}U = \dfrac{1}{\phi_1}\dfrac{du}{d\eta},\ \dfrac{d\theta}{d\eta} = \phi_2\dfrac{dT}{d\eta}$

at $\quad \eta = -a;\ U = 0,\ T = 0 \qquad (20)$

where,

$$s=\sqrt{\frac{1}{K\phi_1}+\frac{M^2}{\phi_1}},\ d=\frac{\beta}{\sqrt{K}},\ K=k_0/h_1^2 \text{ is the permeability parameter, } M=\sqrt{\frac{\sigma}{\mu}}B_0h_1$$

is the Hartmann number, $N_{Br}=\mu V^{*2}/k(T^*-T_0)$ is the Brinkman number, and $R=\dfrac{kk'}{4\sigma' T_0^3}$ is the radiation Parameter.

The solution of equations (16), (17), (18) and (19) under the boundary conditions (20) for non-dimensional velocity and temperature profiles are as follows:

$$u=A_1e^{M\eta}+A_2e^{-M\eta} \tag{21}$$

$$U=A_3e^{s\eta}+A_4e^{-s\eta} \tag{22}$$

$$\theta=-\frac{3RN_{Br}}{2(3R+4)}(A_1^2e^{2M\eta}+A_2^2e^{-2M\eta})+A_5\eta+A_6 \tag{23}$$

$$T=-\frac{3RN_{Br}\phi_1}{2(3\phi_2R+4)}(A_3^2e^{2s\eta}+A_4^2e^{-2s\eta})+A_7\eta+A_8 \tag{24}$$

where,

$$A_1=\frac{M(e^{2sa}-1)+D}{D_1},\ A_2=e^M(1-A_1e^M),\ A_3=\frac{2MA_1e^{2sa}}{M(e^{2sa}-1)+D}$$

$$A_4=\frac{-2MA_1}{M(e^{2sa}-1)+D}$$

$$D=e^{2sa}(s\phi_1-d)+(s\phi_1+d),\ D_1=M(e^{2as}-1)(e^M+e^{-M})+D(e^M-e^{-M})$$

$$d_1=1+\frac{3RN_{Br}}{2(3R+4)}(A_1^2e^{2M}+A_2^2e^{-2M})$$

$$d_2=\frac{3RN_{Br}}{2}\left(\frac{A_1^2+A_2^2}{3R+4}-\phi_1\left(\frac{A_3^2+A_4^2}{3\phi_2R+4}\right)\right)$$

$$d_3=3RN_{Br}\left(\frac{M(A_1^2-A_2^2)}{3R+4}-s\phi_1\left(\frac{A_3^2-A_4^2}{3\phi_2R+4}\right)\right)$$

$$d_4=\frac{3RN_{Br}\phi_1(A_3^2e^{-2sa}+A_4^2e^{2sa})}{2(3\phi_2R+4)}$$

$$A_5=d_3+\phi_2A_7,\ A_6=d_1-A_5,\ A_7=\frac{(d_1-d_2-d_3-d_4)}{(a+\phi_2)},\ A_8=A_6-A_2$$

The non-dimensional rate of heat transfer at the upper impermeable plate is given by

$$\left(\frac{d\theta}{d\eta}\right)_{\eta=1}=\theta'(1)=-\frac{3RN_{Br}M(A_1^2e^{2M}-A_2^2e^{-2M})}{(3R+4)}+A_5 \tag{25}$$

The non-dimensional rate of heat transfer at the lower impermeable bottom is given by

$$\left(\frac{dT}{d\eta}\right)_{\eta=-a} = T'(-a) = -\frac{3RN_{Br}s\phi_1\left(A_3^2e^{-2sa} - A_4^2e^{2sa}\right)}{(3\phi_2R+4)} + A_7 \tag{26}$$

The non-dimensional shear stress at the upper impermeable plate is given by

$$\left(\frac{du}{d\eta}\right)_{\eta=1} = u'(1) = A_1Me^{M} - A_2Me^{-M} \tag{27}$$

The non-dimensional shear stress at the lower impermeable bottom is given by

$$\left(\frac{dU}{d\eta}\right)_{\eta=-a} = U'(-a) = s\left(A_3e^{-as} - A_4e^{as}\right) \tag{28}$$

3. Entropy Generation

In all sort of thermal engineering applications, heat transfer irreversibility and fluid friction irreversibility are common. Following Bejan, the volumetric rate of entropy generation for both porous and clear fluid regions by considering viscous dissipation effect, can be written as

For clear fluid region:

$$(S''')_1 = \frac{k}{T_r^2}\left[\left(\frac{dt_1}{dy}\right)^2 + \frac{16\sigma'T_0^3}{3kk'}\left(\frac{dt_1}{dy}\right)^2\right] + \frac{1}{T_r}\left\{\mu\left(\frac{du_1}{dy}\right)^2\right\} + \frac{\sigma B_0^2u_1^2}{T_r}$$

For porous region:

$$(S''')_2 = \frac{\bar{k}}{T_r^2}\left[\left(\frac{dt_2}{dy}\right)^2 + \frac{16\sigma'T_0^3}{3\bar{k}k'}\left(\frac{dt_2}{dy}\right)^2\right] + \frac{1}{T_r}\left\{\frac{\mu u_2^2}{k_0} + \bar{\mu}\left(\frac{du_2}{dy}\right)^2\right\} + \frac{\sigma B_0^2u_2^2}{T_r}$$

where, T_r is the reference temperature

The dimensionless form of the entropy generation number in the clear fluid region (I) is given by

$$(Ns)_1 = \frac{(S''')_1}{S_0'''} = (HTI)_1 + (FFI)_1 + (MFI)_1 \tag{29}$$

where,

$S_0''' = k(T^* - T_0)^2/T_r^2h_1^2$, is the reference volumetric entropy generation;

$T' = T' = T_r/(T^* - T_0)$, is the non-dimensional reference temperature;

$(HTI)_1 = \left(1+\frac{4}{3R}\right)\left(\frac{d\theta}{d\eta}\right)^2$, is the heat transfer irreversibility in clear fluid region;

$(FFI)_1 = T'N_{Br}\left(\frac{du}{d\eta}\right)^2$, is the fluid friction irreversibility in clear fluid region.

$(\mathrm{MFI})_1 = \mathrm{T'N_{Br}M^2}u^2$, is the magnetic friction irreversibility in clear fluid region.

The dimensionless form of the entropy generation number in porous region (II) is given by

$$(\mathrm{Ns})2 = \frac{(\mathrm{S'''})_2}{\mathrm{S'''_0}} = (\mathrm{HTI})_1 + (\mathrm{FFI})_1 + (\mathrm{MFI})_1 \tag{30}$$

where,

$(\mathrm{HTI})_2 = \left(\phi_2 + \dfrac{4}{3\mathrm{R}}\right)\left(\dfrac{d\mathrm{T}}{d\eta}\right)^2$ is the heat transfer irreversibility in porous region

$(\mathrm{FFI})_2 = \mathrm{T'N_{Br}}\left[\dfrac{\mathrm{U}^2}{\mathrm{K}} + \phi_1\left(\dfrac{d\mathrm{U}}{d\eta}\right)^2\right]$ is the fluid friction irreversibility in porous region

$(\mathrm{MFI})_2 = \mathrm{T'N_{Br}M^2U^2}$ is the magnetic friction irreversibility in porous region

In many engineering designs and optimization problems, the contribution of the heat transfer entropy generation to the total entropy generation rate is required; therefore, Paoletti et. al., [44] presented Bejan number (Be) which is an alternative irreversibility distribution parameter in terms and defined as the ratio of the entropy generation due to heat transfer ($\mathrm{N_H}$) to the total entropy generation (Ns). Bejan number is given by the following mathematical expression

For clear fluid region:

$$(\mathrm{Be})_1 = \frac{(\mathrm{HTI})_1}{(\mathrm{Ns})_1}$$

For porous region:

$$(\mathrm{Be})_2 = \frac{(\mathrm{HTI})_2}{(\mathrm{Ns})_2}$$

Analyzing the expression of Bejan number, it could be observed that its value lies between $0 < \mathrm{Be} < 1$ with the following extreme cases and when entropy is generated by heat transfer irreversibility and fluid friction irreversibility, the value of Bejan number could be Be = 1 and Be = 0 respectively.

4. Figures and Discussion

Figures 2-3 represent the fluid temperature profiles across the channel. Generally, the fluid temperature is zero at the lower fixed plate and gradually increasing within the channel toward the upper moving plate. As M increases, due to a rise in magnetic field intensity, a further increase in the fluid temperature is observed. This can be attributed to the increasing effect of Joule heating. It also shows the temperature profiles for different values of R. The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is obvious that an increase in the radiation parameter results in increasing temperature throughout the channel. However the effect of velocity slip parameter β is to reduce the temperature in the channel.

The effect of viscosity ratio parameter ϕ_1 on the temperature is shown in Figure 3. From this figure, we observe that when the value of viscosity ratio parameter increases, the temperature distribution also increases in the channel. Meanwhile, a fall in the fluid temperature is observed with an increase in permeability parameter K. It is seen that the temperature field increases with the increase in the value of the Brinkman number N_{Br} because viscous forces generates more energy to enhance fluid temperature in the channel, and it is already known that N_{Br} accounts for the relevance of viscous heating.

Figures 4-5 illustrate the effect of thermo-physical parameters on rate of heat transfer at the upper moving plate. It can be seen that the rate of heat transfer decreases at the upper moving plate with the increase in the N_{Br} value, become zero at certain critical brinkman number N_{Br}^* then changes sign and further increases in magnitude. The value of critical brinkman number further increases by increasing the value of β, K and decreases by increasing the value of ϕ_1, M, R.

Figures 6-7 show the influence of physical parameters on the local entropy generation number Ns plotted against η. It is seen that the entropy generation number *Ns* is low in the middle part of the channel because of gradually varying small temperature gradient there, and attains high values in the vicinity of the upper moving wall.

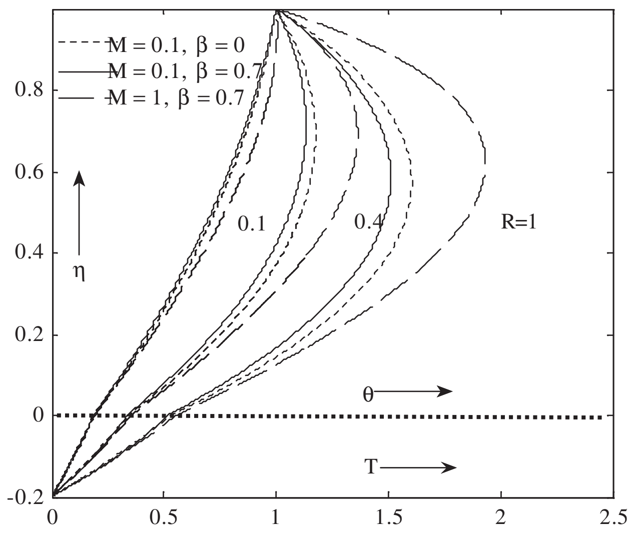

Figure 2: Temperature distribution for
$K = 0.1$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$, $N_{Br} = 20$

It is more pronounced in the region near the upper wall of the channel because of high temperature gradient there. It is seen that the Brinkman number enhances the entropy generation number throughout the channel and the effect of velocity slip parameter is to decrease the total entropy in the clear fluid region, while reverse effect has been observed in the porous region. Further an increment in permeability parameter K, viscosity ratio ϕ_1 and magnetic field parameter M increases the

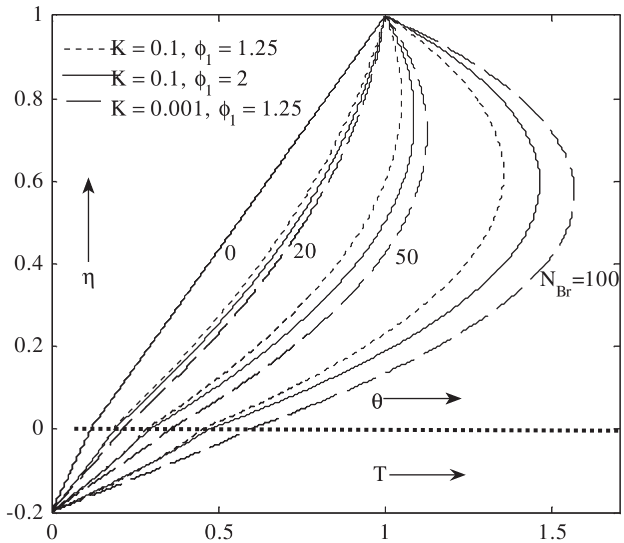

Figure 3. Temperature distribution for M = 0.1, R = 0.1, ϕ_2 = 1.5, a = 0.2, β = 0.7

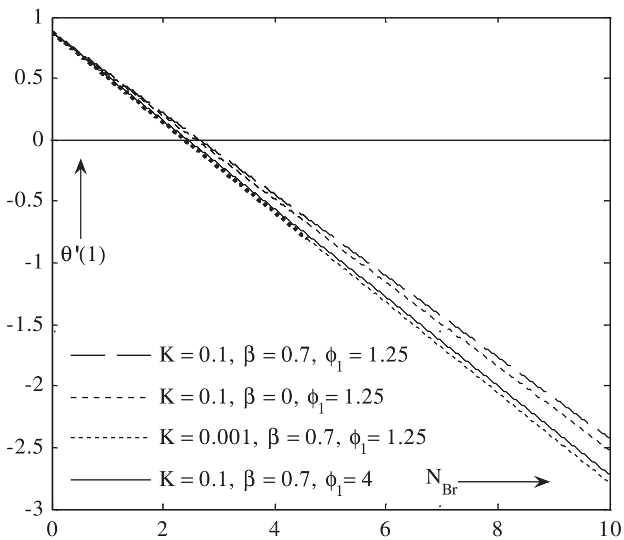

Figure 4. Rate of heat transfer at the upper wall for M = 1, R = 1, ϕ_2 = 1.5, a = 0.2

entropy generation number in the clear fluid region and the effects are reversed in the porous region. The effect of radiation parameter R is to increase the entropy generation in the channel except the middle part of the channel where it decreases by increasing R.

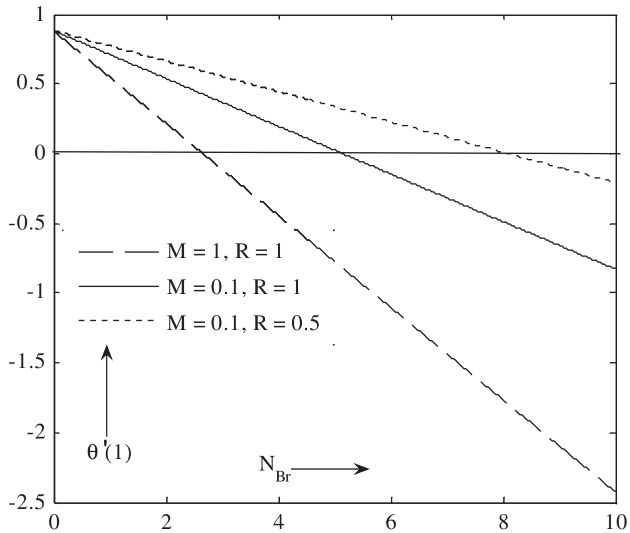

Figure 5. Rate of heat transfer at the upper wall for $K = 0.1$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$, $\beta = 0.7$

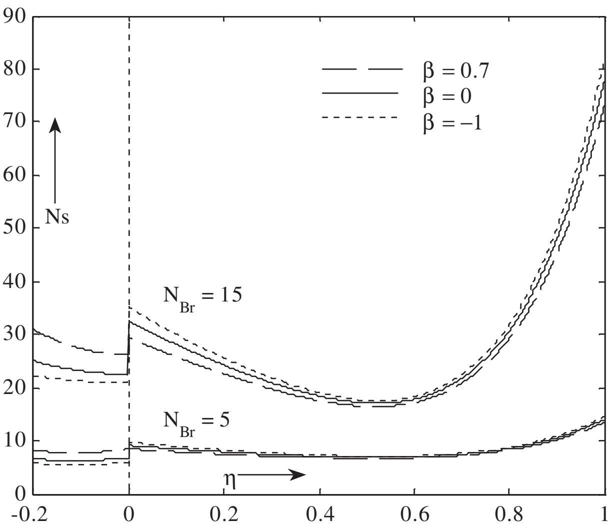

Figure 6. Total entropy generation Ns vs. η for $K = 0.1$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$, $M = 1$, $R = 1$

To get an idea of whether the fluid friction and magnetic field irreversibility dominates over the heat transfer or vice versa, the Bejan number Be is introduced. From Figures 8-9, it can be seen that for some moderate values of Brinkman number,

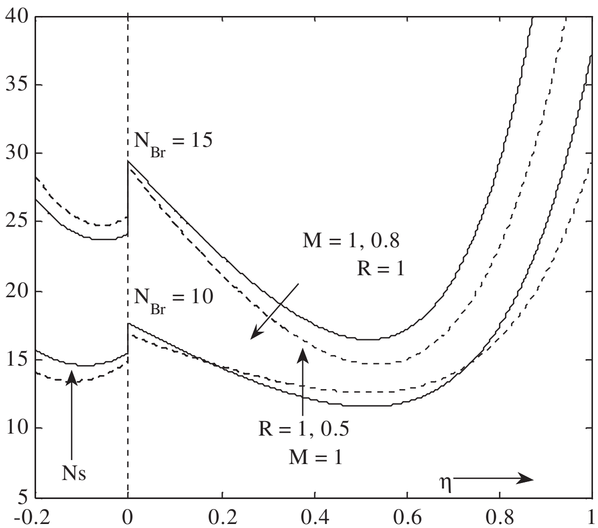

Figure 7. Total entropy generation Ns vs. η for $K = 0.1$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$, $\beta = 0.7$

Bejan number is less than 0.5 in the upper half part of the channel, which shows that the irreversibility effects due to fluid friction and magnetic field dominates in that region, while the heat transfer irreversibility effects dominates at the lower fixed plate. The Bejan number increases at both plates and at fluid-porous interface by increasing the Brinkman number, while the reverse effects have been observed in the middle of the channel. The minimum value of Bejan number occurs somewhere near the upper moving plate, which shifts towards the middle of the channel by increasing the value of Brinkman number. The effect of velocity slip parameter on Bejan number is significant only in the porous region, where it decreases on increasing the velocity slip parameter. Further the effect of increasing magnetic field parameter M is to increase the Bejan number everywhere in the channel, consequently, the heat transfer irreversibility effects become dominant in the channel. The situation is reversed with increasing radiation parameter R, except the region near the upper moving plate. The Bejan number decreases at the lower fixed plate region due to a rise in radiation parameter and irreversibility due to fluid friction and magnetic field become dominant. At the upper moving plate, the Bejan number increases due to an increase in radiation parameter, consequently, heat transfer irreversibility become dominant.

The effect of various parameters on entropy generation due to magnetic field is shown in Figures 10-12. Figure 10 show that the entropy generation due to magnetic friction decreases near the plates on increasing the Brinkman number N_{Br}, while the reverse effects have been observed in the middle of the channel. Further the entropy due to magnetic field increases on increasing the velocity slip parameter β or magnetic field parameter M. The radiation parameter R decreases the entropy due to magnetic field near the upper moving plate and the effect is reversed in the middle of the channel. The effect of radiation parameter is not significant in the porous region and near the clear fluid-porous interface.

Figures 13-15 presents the effect of various parameters on entropy generation due to fluid friction. Figure 13 show that the entropy generation due to fluid friction decreases near the plates on increasing the Brinkman number N_{Br}, while the reverse effects have been observed in the middle of the channel. In Figure 14, we can be seen that the increasing value of magnetic field parameter M decreases the entropy due to fluid friction throughout the channel, while the increasing value of radiation parameter R decreases the entropy due to fluid friction near the plates and the effect is reversed in the middle of the channel. Further the entropy due to

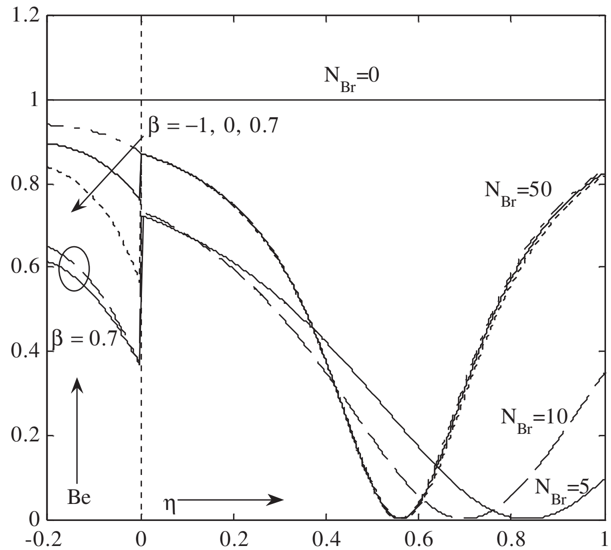

Figure 8. Bejan no. vs. η for $K = 0.1$, $\phi_2 = 1.5$, $a = 0.2$, $R = 1$, $\phi_1 = 1.25$, $M = 1$

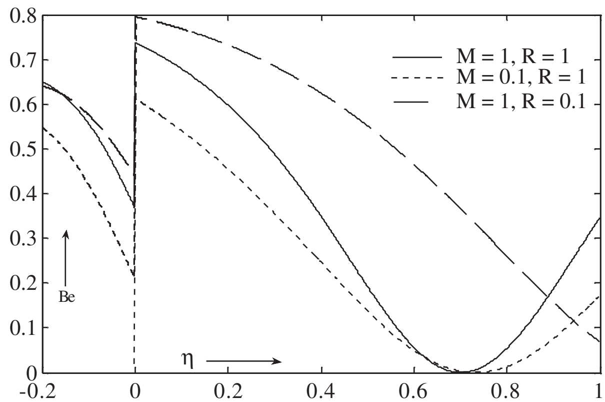

Figure 9. Bejan no. vs. η for $K = 0.1$, $\phi_2 = 1.5$, $a = 0.2$, $\beta = 0.7$, $\phi_1 = 1.25$, $N_{Br} = 10$

fluid friction decreases in the clear fluid region with the increase in velocity slip parameter β, while the effect are reversed in the porous region, where it increases with the increase in velocity slip parameter as shown in Figure 15.

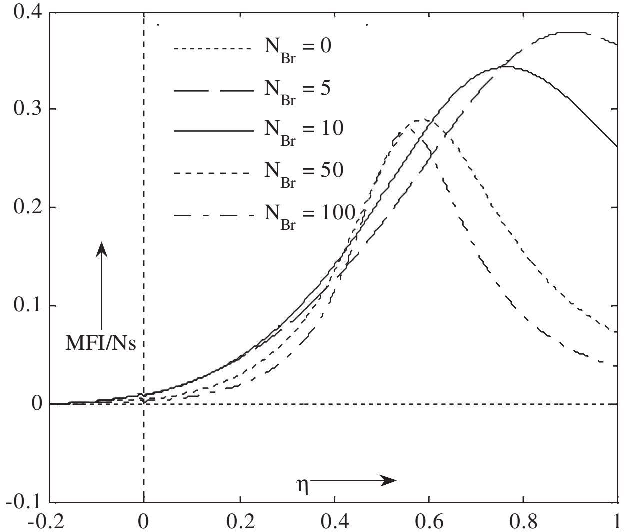

Figure 10. MFI/Ns vs. η for K = 0.1, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$, M = 1, R = 1, $\beta = 0.7$

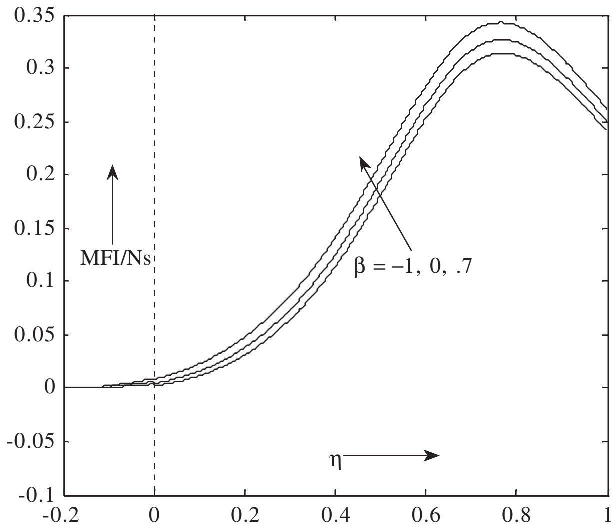

Figure 11. MFI/Ns vs. η for K = 0.1, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$, M = 1, R = 1, $N_{Br} = 10$

5. Conclusions

This paper analyzed the inherent irreversibility in a MHD shear driven flow through a channel partially filled with porous medium in the presence of radiation and viscous dissipation. Exact solutions have been found for the governing equations. Some of the results obtained can be summarized as follows:

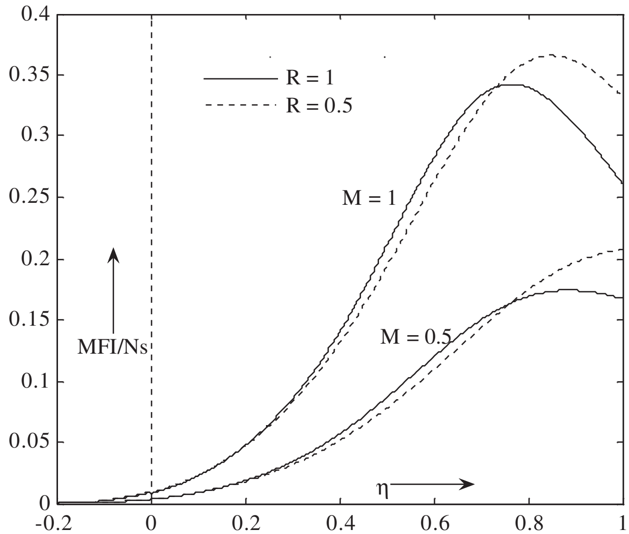

Figure 12. MFI/Ns vs. η for K = 0.1, ϕ_1 = 1.25, ϕ_2 = 1.5, *a* = 0.2, N_{Br} = 10, β = 0.7

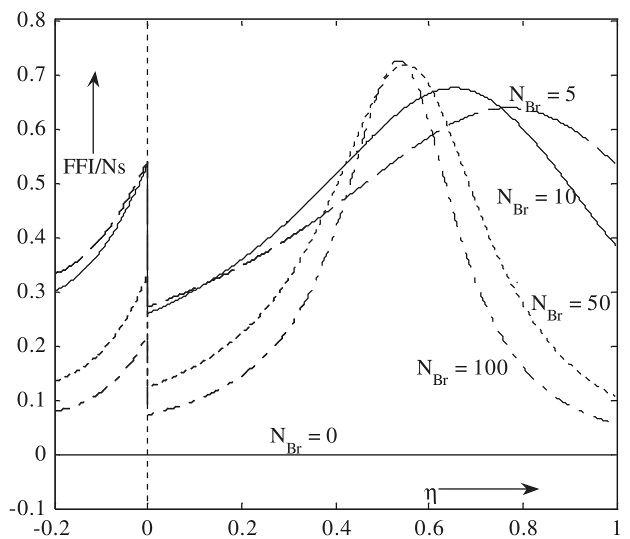

Figure 13. FFI/Ns vs. η for K = 0.1, ϕ_1 = 1.25, ϕ_2 = 1.5, *a* = 0.2, M = 1, R = 1, β = 0.7

- Increment in R, ϕ_1 & N_{Br} increase the temperature throughout the channel while increment in K reduces the fluid temperature and when M increase a further increment in the clear fluid temperature is observed but it shows reverse effect in partially porous fluid.

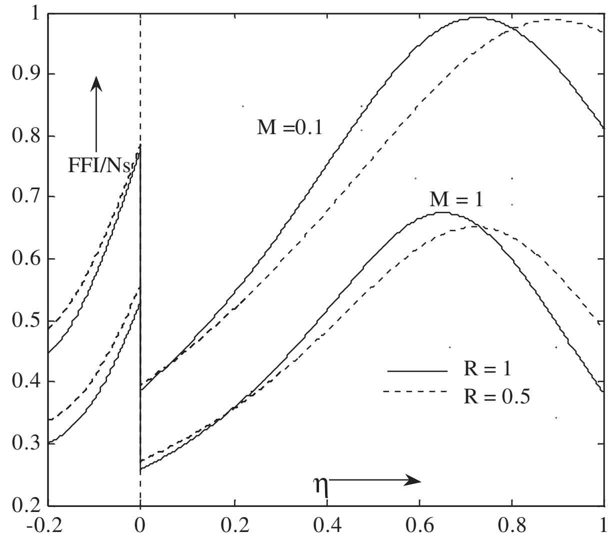

Figure 14. FFI/Ns vs. η for K = 0.1, ϕ_1 = 1.25, ϕ_2 = 1.5, a = 0.2, N_{Br} = 10, β = 0.7

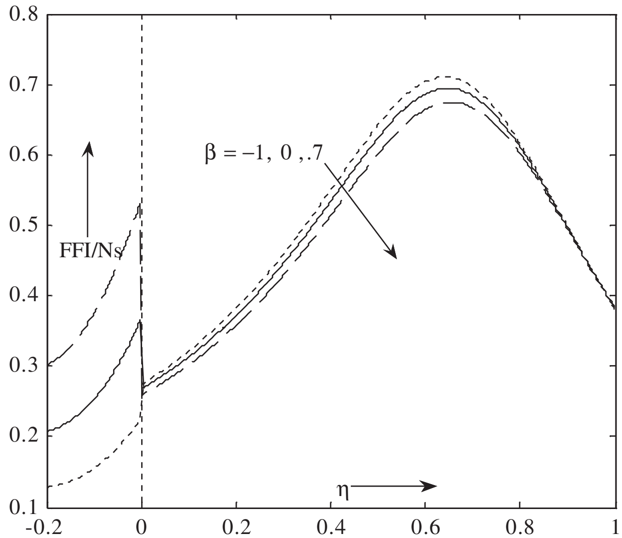

Figure 15. FFI/Ns vs. η for K = 0.1, ϕ_1 = 1.25, ϕ_2 = 1.5, a = 0.2, M = 1, R = 1, N_{Br} = 10

- The rate of heat transfer decreases at the upper moving plate with the increase in the N_{Br} value, The value of critical brinkman number further increases by increasing the value of β, K and decreases by increasing the value of ϕ_1, M, R.
- Ns increases with the increment of N_{Br} & R in the channel except the middle part of the channel where it decreases and Ns enhance in clear fluid region

when M, ϕ_1 & K increases while decreases in partially porous region, it shows reverse trend for velocity slip parameter β for both the regions.

- The increment in Be has been observed when M & N_{Br} increases but for N_{Br} it shows reverse effect in middle of the channel and it decreases with the increase value of R & β but β is significant only in porous region.
- Entropy due to magnetic friction & fluid friction decreases near the plates on increasing the Brinkman number N_{Br} & R while the reverse effects have been observed in the middle of the channel and an increment of M & β enhance entropy due to magnetic friction while decreases due to fluid friction except entropy due to fluid friction in porous region increases by the increasing value of β.

From the following observations, it is concluded that the optimal design and the efficient performance of a flow system or a thermally designed system can be improved by choosing the appropriate values of the physical parameters.

6. Conflict of Interest

There is no conflict of interest in publication of this article.

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