AN INVENTORY MODEL FOR WEIBULL DETERIORATING ITEMS WITH EXPONENTIAL DEMAND AND TIME-VARYING HOLDING COST

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Abstract. This paper develops a deterministic inventory model for those inventory systems where stored inventory deteriorates continuously with weibull distribution. It is assumed that the demand rate is exponential with time and the time-varying holding cost is linear. This model is developed without allowing shortages. To check the effect of different parameters the sensitivity analysis is carried out with an example.

1. Introduction

Generally, deterioration is considered as spoilage, damage, devaluation, evaporation etc. Deterioration of physical goods in stock is a real characteristic. Fruits, vegetables, meat, foodstuffs, gasoline, fashion and seasonal goods, blood, chemicals, medicines, radioactive substances, photographic films, etc., deteriorate during their usual storage period. So it is necessary to develop optimal inventory policies to control and maintain different kind of deteriorating items.


In present paper a deterministic economic order quantity model is derived considering two parameter weibull deterioration rate with exponential demand function and linear holding cost.

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Key words and phrases. Inventory, Deteriorating item, Weibull, Exponential.
2. Assumptions

- Two parameter weibull deterioration rate.
- Demand rate is exponential function of time.
- Holding cost is a linear function of time.
- Shortages are not permitted.
- Lead time is negligible.
- Instant and infinite replenishment rate.

3. Notations

- \( D(t) = ke^{\gamma t} \) is demand function; \( 0 \leq \gamma < 1 \).
- \( I(t) \) = Inventory level at any instant of time \( t; 0 \leq t \leq T \).
- \( A \) = Ordering cost per order.
- \( C_d \) = Deterioration cost per unit (purchase price minus salvage value).
- \( h(t) = x + yt \), inventory variable holding cost.
- \( \alpha \beta t^{\beta-1} \) = The two parameter weibull deterioration rate; \( 0 \leq \alpha \leq 1, \beta > 0 \)

4. Model development

The inventory level will continuously decrease with time due to demand and deterioration. The change of inventory level can be described by the below differential equation.

\[
\frac{dI(t)}{dt} + \alpha \beta t^{\beta-1} I(t) = -ke^{\gamma t}; 0 \leq t \leq T
\]  

(4.1)

Solution of equation (4.1) using boundary condition \( I(T) = 0 \) is

\[
I(t) = k[(T-t) + \gamma \frac{(T^2 - t^2)}{2} + \frac{\alpha(T^{\beta+1} - t^{\beta+1})}{\beta + 1} + \frac{\alpha \gamma (T^{\beta+2} - t^{\beta+2})}{\beta + 2} + \alpha t^{\beta}(t-T)]
\]  

(4.2)

Initial order quantity at \( t = 0 \) is

\[
I_0 = I(0) = k[T + \gamma \frac{T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta + 1} + \frac{\alpha \gamma T^{\beta+2}}{\beta + 2}]
\]  

(4.3)

Ordering cost per unit time is

\[
OC = \frac{A}{T}
\]  

(4.4)

Total demand during the cycle period \([0, T]\) is

\[
\int_0^T D(t)dt = \int_0^T ke^{\gamma t}dt = \frac{k}{\gamma}[e^{\gamma T} - 1]
\]  

(4.5)

The number of deteriorated units in \([0, T]\) is

\[
I_0 - \int_0^T D(t)dt = k[T + \gamma \frac{T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta + 1} + \frac{\alpha \gamma T^{\beta+2}}{\beta + 2}] - \frac{k}{\gamma}[e^{\gamma T} - 1]
\]  

(4.6)

Cost due to deterioration per unit time is

\[
DC = \frac{kC_d}{T}[T + \gamma \frac{T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta + 1} + \frac{\alpha \gamma T^{\beta+2}}{\beta + 2}] - \frac{kC_d}{\gamma T}[e^{\gamma T} - 1]
\]  

(4.7)
Inventory variable holding cost per unit time is
\[ HC = \frac{1}{T} \int_0^T (x + yt)I(t)dt \]
\[ = \frac{xk}{T} \frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha T^{\beta+2}}{\beta + 1} + \frac{\alpha T^{\beta+3}}{\beta + 3} + \frac{\alpha T^{\beta+4}}{2(\beta + 4)} \]
\[ + \frac{yk}{T} \left[ \frac{T^3}{6} + \frac{\gamma T^4}{8} + \frac{\alpha T^{\beta+3}}{2(\beta + 2)(\beta + 3)} + \frac{\alpha T^{\beta+4}}{2(\beta + 4)} \right] \] (4.8)

Total variable inventory cost per unit time is
\[ TC(T) = OC + DC + HC \]
\[ = \frac{A}{T} + \frac{kC_d}{T} \left[ T + \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta + 1} + \frac{\alpha T^{\beta+2}}{\beta + 2} \right] - \frac{kC_d}{\gamma T} \left( e^{\gamma T} - 1 \right) \]
\[ + \frac{xk}{T} \left[ \frac{T^2}{2} + \frac{\gamma T^3}{3} + \frac{\alpha T^{\beta+2}}{\beta + 2} \right] \]
\[ + \frac{yk}{T} \left[ \frac{T^3}{6} + \frac{\gamma T^4}{8} + \frac{\alpha T^{\beta+3}}{2(\beta + 2)(\beta + 3)} \right] \] (4.9)

Our aim is to minimize the total variable inventory cost per unit time, the necessary and sufficient conditions to minimize total cost \( TC(T) \) for a given value \( T \) are
\[ \frac{\partial TC(T)}{\partial T} = \frac{A}{T^2} + kC_d \left[ \frac{\gamma T^{\beta+1}}{\beta + 1} + \frac{\alpha T^{\beta+2}}{\beta + 2} \right] - \frac{kC_d}{\gamma T^2} \left( e^{\gamma T} - e^{\gamma T} + 1 \right) \]
\[ + \frac{xk}{2} \left[ \frac{\gamma T^2}{2} + \frac{\alpha T^{\beta+1}}{\beta + 2} + \frac{\alpha T^{\beta+2}}{\beta + 3} \right] \]
\[ + \frac{yk}{3} \left[ \frac{3\gamma T^2}{8} + \frac{\alpha T^{\beta+1}}{2(\beta + 3)} + \frac{\alpha T^{\beta+2}}{2(\beta + 4)} \right] = 0. \] (4.10)

\[ \frac{\partial^2 TC(T)}{\partial T^2} = \frac{A}{T^3} + kC_d \left[ \frac{\alpha T^{\beta+2}}{\beta + 1} + \frac{\alpha T^{\beta+3}}{\beta + 2} \right] \]
\[ - \frac{kC_d}{\gamma T^2} \left( \frac{T e^{\gamma T} - \gamma e^{\gamma T}}{T^2} \right) - \frac{\gamma}{T^4} \left( \frac{T e^{\gamma T} - 2T e^{\gamma T}}{T^4} - \frac{2}{T^3} \right) \]
\[ + \frac{xk}{2} \left[ \frac{\gamma T^2}{3} + \frac{\alpha T^{\beta+1}}{\beta + 2} + \frac{\alpha T^{\beta+2}}{\beta + 3} \right] \]
\[ + \frac{yk}{4} \left[ \frac{3\gamma T^2}{4} + \frac{\alpha T^{\beta+1}}{2(\beta + 3)} + \frac{\alpha T^{\beta+2}}{2(\beta + 4)} \right] > 0. \] (4.11)

5. Numerical Example and Sensitivity Analysis

5.1. Example-1. In the above developed model, consider the following values of different parameters in proper units.
\( A = 500, C_d = 15, \alpha = 0.04, \beta = 2, k = 250, \gamma = -0.02, x = 5, y = 0.05. \)

Solving equation (4.9) in R programming with the above values of parameters, we obtain the optimal cycle length \( T^* = 0.837232 \) and from equation (4.3) and (4.9), the optimal order quantity \( Q^* = 209.487252 \) and the optimal total inventory cost \( TC^* = 1155.314. \)
5.2. Sensitivity Analysis. Now we study the effect of changes in values of parameters $A, \alpha, \beta, k, \gamma, x$ and $y$ with Example-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% Change</th>
<th>$T^*$</th>
<th>$Q^*$</th>
<th>$TC^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>+50%</td>
<td>1.009976</td>
<td>253.33466</td>
<td>1425.798</td>
</tr>
<tr>
<td>$A$ 600</td>
<td>+20%</td>
<td>0.911251</td>
<td>228.23018</td>
<td>1269.683</td>
</tr>
<tr>
<td>400</td>
<td>-20%</td>
<td>0.754205</td>
<td>188.54582</td>
<td>1029.665</td>
</tr>
<tr>
<td>250</td>
<td>-50%</td>
<td>0.603840</td>
<td>150.77671</td>
<td>808.9180</td>
</tr>
<tr>
<td>0.06</td>
<td>+50%</td>
<td>0.812102</td>
<td>204.02205</td>
<td>1174.388</td>
</tr>
<tr>
<td>$\alpha$ 0.048</td>
<td>+20%</td>
<td>0.826713</td>
<td>207.20167</td>
<td>1163.092</td>
</tr>
<tr>
<td>0.032</td>
<td>-20%</td>
<td>0.848469</td>
<td>211.92560</td>
<td>1147.321</td>
</tr>
<tr>
<td>0.02</td>
<td>-50%</td>
<td>0.866897</td>
<td>215.91716</td>
<td>1134.884</td>
</tr>
<tr>
<td>3</td>
<td>+50%</td>
<td>0.839592</td>
<td>209.36129</td>
<td>1141.236</td>
</tr>
<tr>
<td>$\beta$ 2.4</td>
<td>+20%</td>
<td>0.837651</td>
<td>209.24816</td>
<td>1148.644</td>
</tr>
<tr>
<td>1.6</td>
<td>-20%</td>
<td>0.838150</td>
<td>210.18216</td>
<td>1164.078</td>
</tr>
<tr>
<td>1</td>
<td>-50%</td>
<td>0.844641</td>
<td>212.90362</td>
<td>1183.654</td>
</tr>
<tr>
<td>-0.01</td>
<td>+50%</td>
<td>0.832727</td>
<td>211.58570</td>
<td>1158.625</td>
</tr>
<tr>
<td>$\gamma$ -0.016</td>
<td>+20%</td>
<td>0.835402</td>
<td>210.31658</td>
<td>1156.653</td>
</tr>
<tr>
<td>-0.024</td>
<td>-20%</td>
<td>0.839101</td>
<td>208.67100</td>
<td>1153.955</td>
</tr>
<tr>
<td>-0.03</td>
<td>-50%</td>
<td>0.841979</td>
<td>207.47011</td>
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<tr>
<td>375</td>
<td>+50%</td>
<td>0.692127</td>
<td>259.39180</td>
<td>1406.233</td>
</tr>
<tr>
<td>$k$ 300</td>
<td>+20%</td>
<td>0.768798</td>
<td>230.66288</td>
<td>1261.862</td>
</tr>
<tr>
<td>200</td>
<td>-20%</td>
<td>0.928626</td>
<td>186.10622</td>
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<tr>
<td>125</td>
<td>-50%</td>
<td>1.151517</td>
<td>144.78302</td>
<td>828.5111</td>
</tr>
<tr>
<td>7.5</td>
<td>+50%</td>
<td>0.703641</td>
<td>175.82148</td>
<td>1393.743</td>
</tr>
<tr>
<td>$x$ 6</td>
<td>+20%</td>
<td>0.775622</td>
<td>193.93878</td>
<td>1255.751</td>
</tr>
<tr>
<td>4</td>
<td>-20%</td>
<td>0.915131</td>
<td>229.20865</td>
<td>1046.177</td>
</tr>
<tr>
<td>2.5</td>
<td>-50%</td>
<td>1.081919</td>
<td>271.70633</td>
<td>860.2226</td>
</tr>
<tr>
<td>0.075</td>
<td>+50%</td>
<td>0.836283</td>
<td>209.24744</td>
<td>1156.040</td>
</tr>
<tr>
<td>$y$ 0.06</td>
<td>+20%</td>
<td>0.836852</td>
<td>209.39123</td>
<td>1155.605</td>
</tr>
<tr>
<td>0.04</td>
<td>-20%</td>
<td>0.837613</td>
<td>209.58353</td>
<td>1155.023</td>
</tr>
<tr>
<td>0.025</td>
<td>-50%</td>
<td>0.838170</td>
<td>209.72430</td>
<td>1154.586</td>
</tr>
</tbody>
</table>
From above table and chart it is clear that as ordering cost $A$ increases, the optimal cycle length $T^*$ increases. As demand parameters, deterioration parameters and carrying cost parameters increases, the optimal cycle length $T^*$ decreases. That is as $k, \gamma, \alpha, \beta, x$ and $y$ increases, $T^*$ decreases.

In above chart we can observe that as ordering cost $A$ and demand parameters $k$ and $\gamma$ increases, the optimal order quantity $Q^*$ increases. Also as deterioration and carrying cost parameters $\alpha, \beta$ and $x$ increases, $Q^*$ decreases.
From table-1 and above chart we can observe that as $A$ increases, that is as ordering cost increases the corresponding total cost increases. Increase in parameter $\alpha$ results in increase in total cost, however increase in shape parameter ($\beta$) of weibull distribution results in decrease in the ordering quantity $Q^*$ and hence total cost also decreases. We can also observe that as $x$ and $y$, the parameters associated with the carrying cost increases, the corresponding total cost increases. Increase in $k$ and $\gamma$, the parameters associated with demand also results in increase in total cost.

5.3. Example-2. $A = 500$, $C_d = 20$, $\alpha = 0.08$, $\beta = 4$, $k = 400$, $\gamma = 0.01$, $x = 10$, $y = 0.05$. Solving equation (4.9) in R programming with the above values of parameters, we obtain the optimal cycle length $T^* = 0.4779953$ and from equation (4.3) and (4.9), the optimum order quantity $Q^* = 195.933769$ and the optimal total variable inventory cost $TC^* = 2038.264$.

6. Conclusion

In present paper we approximated optimal inventory ordering policies for the items having Weibull deterioration rate and exponential demand. It is supposed that the carrying cost is linear function of time. The sensitivity analysis clearly shows increase/decrease in the value of cost with the corresponding increase/decrease in the parameter values.
References


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