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A SEGMENTATION ALGORITHM USING ε-NEIGHBOURHOODS

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ABSTRACT

Image segmentation plays an important role in image processing. Medical image segmentation is useful in diagnosis and treatment of diseases. One of the methods of image segmentation is region growing method. Here, we developed a new seeded region growing segmentation algorithm based on ε - neighbours of different metrics and grouping criterion for segmentation of region of interest from MRIs. The qualities of segmented images are measured by the evaluation measure "Accuracy ".

KEYWORDS

Metrics, Topological Neighbourhoods, Region Growing, Segmentation.

1991 Mathematics Subject Classification Primary 54A99, Secondary 54E35.

1 INTRODUCTION

Image segmentation is a process that partitions a digital image into disjoint connected sets of pixels, each of which corresponds to an object or region. Medical image segmentation plays an important role in diagnosis and treatment planning for the radiologists. Image segmentation algorithms are mainly based on two properties, detecting discontinuities and similarities. The first category is based on the abrupt changes in intensity and the second category is based on grouping the set of similar pixels satisfying predefined criterion. The image segmentation algorithms based on the above techniques are edge detection, thresholding, region growing, region splitting and merging, clustering, watershed algorithm etc. In region growing segmentation approach, initially a seed point is selected and the neighbouring pixels satisfying the similarity properties are added to the seed point to get the segmented region.

Andras Hajdu et al.[2] showed how the special family of distance functions generated by neighbourhood sequences can be applied successfully to image segmentation beside classical metric. N. Bonnet et al. [3] gave a 'no-threshold' histogram based image segmentation method which depends on the estimation of global probability density function and the partitioning obtained using tools originating from mathematical morphology. Pastore, J. et al. [9] described an automatic segmentation of computerized axial tomography images with tumors by means of alternating sequential filters of mathematical morphology and connected components extraction based on continuous topology concepts. Gnanambal Ilango and R. Marudhachalam [4] presented an automatic method relevant to medical image segmentation by means of fuzzy hybrid filter for denoising and topological concepts to extract connected components and edge detection. An automatic seeded region growing algorithm to automatically segment 2D cellular images based on pixel intensity and position which overcomes the explosion, under segmentation and over segmentation problems was proposed by M.M. Abdelsamea [1].

Here a new seeded region growing segmentation algorithm is proposed based on ε - neighbours of different metrics and grouping criterion. This work is organized as follows: Section 2 explains the Seeded region growing method of segmentation, Section 3 gives the basic definitions, Section 4 provides the proposed segmentation algorithm, Section 5 explains the method of evaluation of segmentation, Section 6 deals with the experimental work, Section 7 discusses the result analysis, Section 8 gives the conclusion.

2 Seeded Region Growing Method of Segmentation

Region based segmentation is a classical method. This method tries to extract the object that is connected based on some predefined criterion. This criterion can be based on the intensity information and edges in the image. One of the region based segmentation methods is Seeded Region Growing Method. This is a procedure that groups pixels in whole image into subregions based on predefined criterion and processed in four steps.

(i) Select a seed pixel in the original image.

(ii) Select a set of similarity criterion such as gray level or colour and set up a stopping rule.

(iii) Grow region by appending to the seed, those neighbouring pixels that have predefined properties similar to seed pixel.

(iv) Stop region growing when no more pixel meet the criterion for inclusion in that region.

Here from the histogram of the image, the gray level of the region of interest is selected. Considering the gray level of the region of interest, a seed point is selected as a point having maximum number of ε - neighbourhoods. For the similarity criterion, a grouping criterion is defined based on metric and gray level difference. The connected region of interest is grown by including the ε-neighbours of the seed point satisfying the grouping criterion with the seed point. The qualities of the segmented region of interest from the magnetic resonance images for the proposed algorithm are compared with the ground truth images and the results are analyzed.

3 BASIC DEFINITIONS

Definition 3.1. [7]
A metric on a set X is a function $d : X \times X \to R$ having the following properties:

(i) $d(x, y) \geq 0, \forall x, y \in X$; equality holds iff $x = y$.

(ii) $d(x, y) = d(y, x), \forall x, y \in X$.

(iii) $d(x, z) \leq d(x, y) + d(y, z), \forall x, y, z \in X$.

Given a metric d on X, the number $d(x, y)$ is called as the distance between x and y in the metric d.

Definition 3.2. [6]
An image may be defined as a two-dimensional function $f(x, y)$, where x and y are spatial (plane) co-ordinates, and the amplitude of f at any pair of co-ordinates (x, y) is called the intensity or gray level of the image at that point. If x, y and the intensity values of f are all finite, discrete quantities, we call the image a digital image. A digital image is composed of a finite number of elements (x, y) each of which has a particular location and value. These elements are called picture elements or pixels.

Definition 3.3. [8]

Let $p = (x_1,y_1), q = (x_2,y_2) \in S^2$. Consider the functions

(i) $d_4 : S \times S \to R$ defined by $d_4(p,q) = |x_1 - x_2| + |y_1 - y_2|$. Then (S^2, d_4) is a metric space. The metric d_4 is called the City-Block metric or Manhattan metric.

(ii) $d_8 : S \times S \to R$ defined by $d_8(p,q) = max\{|x_1 - x_2|, |y_1 - y_2|\}$. Then (S^2, d_8) is a metric space. The metric d_8 is called the Chessboard metric.

Definition 3.4. [5]

For any metric space $[S^2, d]$, any $p \in S^2$, and any $\varepsilon > 0$, consider the set $N_{d,\varepsilon}(p) = \{q \in S^2; d(p,q) < \varepsilon + 1\}$. $N_{d,\varepsilon}(p)$ is called the (open) ε-neighbourhood of p in S^2.

Definition 3.5. [5]

For any point p, the 4-neighbourhood of p denoted by $N_{d_4,1}(p)$ is defined as $N_{d_4,1}(p) = \{q \in S^2; d_4(p,q) < 2\}$. $N_{d_4,1}(p)$ is also denoted by $N_4(p)$ and the 8-neighbourhood of p denoted by $N_{d_8,1}(p)$ is defined as $N_{d_8,1}(p) = \{q \in S^2; d_8(p,q) < 2\}$. $N_{d_8,1}(p)$ is also denoted by $N_8(p)$.

Definition 3.6. [5]

For any point p, the 4-neighbours of p are $N_{d_4,1}(p) - \{p\}$ and the 8-neighbours of p are $N_{d_8,1}(p) - \{p\}$.

Definition 3.7. [5]

Let $M \subseteq S \times S$ where $M = \{(x, y \pm k); where\ k = 0, 1, 2, 3, ...\}$. Let $p = (x_1,y_1),\ q = (x_2,y_2) \in M$. Consider the function $d_V : M \to R$ defined by $d_V(p,q) = |x_1 - x_2| + |y_1 - y_2|$. (M, d_V) is a metric space. For any point p, the V_ε neighbourhood of p is defined as $N_{d_V,\varepsilon}(p) = \{q; d_V(p,q) < \varepsilon + 1\}$. Hence the V_1 neighbourhood of p is defined as $N_{d_V,1}(p) = \{q; d_V(p,q) < 2\}$ and the V_2 neighbourhood of p is $N_{d_V,2}(p) = \{q; d_V(p,q) < 3\}$. The V_1 neighbours of p are $N_{d_V,1}(p) - \{p\}$ and the V_2 neighbours of p are $N_{d_V,2}(p) - \{p\}$.

Definition 3.8. [5]

Let $M \subseteq S \times S$ where $M = \{(x \pm k, y); where\ k = 0, 1, 2, 3, ...\}$. Let $p = (x_1,y_1), q = (x_2,y_2) \in M$. Consider the function $d_H : M \to R$ defined by $d_H(p,q) = |x_1 - x_2| + |y_1 - y_2|$. (M, d_H) is a metric space. For any point p, the ε-H neighbourhood of p is defined as $N_{d_H,\varepsilon}(p) = \{q; d_H(p,q) < \varepsilon + 1\}$. Hence the 1-$H$ neighbourhood of p is defined as $N_{d_H,1}(p) = \{q; d_H(p,q) < 2\}$ and the 2-H neighbourhood of p is defined as $N_{d_H,2}(p) = \{q; d_H(p,q) < 3\}$. The 1-$H$ neighbours of p are $N_{d_H,1}(p) - \{p\}$ and the 2-H neighbours of p are $N_{d_H,2}(p) - \{p\}$.

Definition 3.9. [5]

Let $M \subseteq S \times S$ where $M = \{\{(x + k, y + k)\} \cup \{(x - k, y - k)\}; where\ k = 0, 1, 2, 3, ...\}$. Let $p = (x_1, y_1), q = (x_2, y_2) \in M$. Consider the function $d_{RT} : M \to R$ defined by $d_{RT}(p,q) = \frac{1}{2}[|x_1 - x_2| + |y_1 - y_2|]$. (M, d_{RT}) is a metric space. For any point p, the RT_ε neighbourhood of p is defined as $N_{d_{RT},\varepsilon}(p) = \{q; d_{RT}(p,q) < \varepsilon + 1\}$. Hence the RT_1 neighbourhood of p is defined as $N_{d_{RT},1}(p) = \{q; d_{RT}(p,q) < 2\}$ and the RT_2 neighbourhood of p is defined as $N_{d_{RT},2}(p) = \{q; d_{RT}(p,q) < 3\}$. $N_{d_{RT},1}(p)$ is also denoted by $R_3(p)$. The RT_1 neighbours of p are $N_{d_{RT},1}(p) - \{p\}$ and the RT_2 neighbours of p are $N_{d_{RT},2}(p) - \{p\}$.

Definition 3.10. Grouping criterion:

Let X be an image in levels of gray and $\Im$ a topology associated with X and let $Y \subset \Im$. Let 'd' be a metric on X. Define $\phi : Y \times X \to R$ such that $\phi((A, x)) = m$ where m is the maximum of the gray level difference between x and the elements of A. Let $A \in Y$. Given a fixed ε and δ, an element $x \in X$ is said to belong to A if $d(x, y) < \varepsilon + 1$ for some $y \in A$ and $\phi((A, x)) < \delta$.

4 PROPOSED SEGMENTATION ALGORITHM

4.1 HVRT ALGORITHM

Let X be an image in levels of gray.

Step I

From the histogram of the image, select the gray level of the region of interest to be segmented.

Step II

From the selected gray level of the region of interest, find the seed point of the region to be segmented. Here, the seed point is a point with maximum number of horizontal neighbours with gray level difference less than δ which is very small. If more than one point has the maximum number of horizontal neighbours, then choose any one of those points as the seed point.

Let $A_1 = \{x\}$ where x is the seed point of the region of interest.

Step III

Choose $\varepsilon = 1$ and $\delta = 2$. If $\exists$ a point $y \in X$ where $y \notin A_1$ such that

$$d_H(x, y) < \varepsilon + 1 \text{ and} \phi((A_1, y)) < \delta. \tag{4.1}$$

then include y in A_1 and rename it as A_2. Repeat Step III again and again for the elements of A_2 till $\exists$ no point $y \in X$ satisfying the condition (4.1). That is, all the horizontal neighbours of elements of A_1 are obtained.

Step IV

If $\exists$ a point $y \in X$ where $y \notin\ A_2$ such that

$$d_V(z, y) < \varepsilon + 1 \text{ where} z \in A_2 \text{ and} \phi((A_2, y)) < \delta. \tag{4.2}$$

then include y in A_2 and rename it as A_3. Repeat Step IV again and again for the elements of A_3 till $\exists$ no point $y \in X$ satisfying the condition (4.2). That is, all the vertical neighbours of elements of A_2 are obtained.

Step V

If $\exists$ a point $y \in X$ where $y \notin\ A_3$ such that

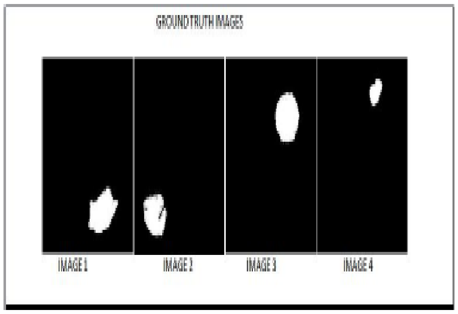
$$d_{RT}(z, y) < \varepsilon + 1 \text{ where} z \in A_3 \text{ and} \phi((A_3, y)) < \delta. \tag{4.3}$$

then include y in A_3 and rename it as A_4. Repeat Step V again and again for the elements of A_4 till $\exists$ no point $y \in X$ satisfying the condition (4.3). That is, all the RT neighbours of elements of A_3 are obtained.

The set A_4 is the segmented region of interest.

5 EVALUATION MEASURE

Let X be the set of pixels in the image. Define the ground truth $T \subset X$ as the set of pixels that were labeled as tumor by the expert. Similarly define the segmented tumor $S \subset\ X$ as the set of pixels that were labeled as tumor by the algorithm. $\bar{T}$ and $\bar{S}$ be the set of pixels that were labeled as non-tumor by the expert and algorithm respectively. The true positive (TP) set is defined as $TP = T \cap S$, i.e., the set of pixels common to T and S. i.e., the set of pixels that were labeled as tumor by the expert and algorithm. The true negative (TN) set is defined as $TN = \bar{T} \cap \bar{S}$. i.e., the set of pixels common to $\bar{T}$ and $\bar{S}$. i.e.,the set of pixels that were



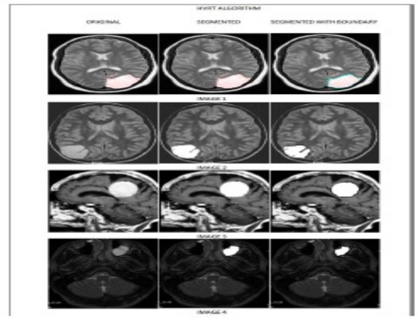


Figure 1: Ground Truth Images

Figure 2: Segmentation by HVRT Algorithm

labeled as non-tumor by the expert and the algorithm. The false negative (FN) set is defined as $FN = T \cap \bar{S}$. i.e., the set of pixels common to T and $\bar{S}$. i.e., the set of pixels that were labeled as tumor by the expert and non-tumor by the algorithm. The false positive (FP) set is defined as $FP = \bar{T} \cap S$. i.e., the set of pixels common to $\bar{T}$ and S. i.e., the set of pixels that were labeled as non-tumor by the expert and tumor by the algorithm.

The segmentation evaluation measure 'Accuracy' is defined as

$$\text{Accuracy} = \frac{n(TP) + n(TN)}{n(TP) + n(TN) + n(FP) + n(FN)}.$$

6 EXPERIMENTAL WORK

In this work MRIs of brain affected by tumor are taken. The seed point of the region of interest is selected using the histogram of the image. The region of interest is grown from the seed point using the metric topological ε- neighbourhoods $N_{d_V,\varepsilon}(p), N_{d_H,\varepsilon}(p), N_{d_{RT},\varepsilon}(p)$ and the grouping criterion. The proposed region growing segmentation algorithm is implemented using MATLAB 7.0. The performance of the proposed algorithm is evaluated using the segmentation evaluation measure 'Accuracy'.

7 RESULT ANALYSIS AND DISCUSSION

The MRIs of brain affected by tumor are segmented using the proposed region growing segmentation algorithm based on metric topological ε-neighbourhoods and the grouping criterion. Table 1 shows the quality of segmentation in % values of the proposed algorithm applied to MRIs of brain affected by tumor. Fig 1 shows the ground truth images. Fig 2 shows the original MRIs of brain affected by tumor, the corresponding segmented region and the segmented region with boundary.

Quality	IMAGE 1	IMAGE 2	IMAGE 3	IMAGE 4
HVRT Algorithm	98.53%	98.89%	99.21%	99.47%

Table-1 Quality of Correct Segmentation in % Values

8 CONCLUSION

In this work a new region growing segmentation algorithm based on metric topological ε-neighbourhoods and grouping criterion is introduced. To demonstrate the performance of the proposed region growing segmentation algorithm based on metric topological ε - neighbourhoods, the experiments have been conducted on MRIs of brain affected by tumor. The performance of the proposed segmentation algorithm is measured using the segmentation evaluation measure "Accuracy". The experimental results indicate that the quality of the correct segmentation is 98.53% and above. This work is a new metric topological approach for the segmentation of medical images.

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