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DETERMINATION OF RELIABILITY SINGLE SAMPLING PLANS BASED ON EXPONENTIATED DISTRIBUTION

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Abstract. Reliability sampling plans are used in manufacturing industries to take decisions on the disposition of lots based on life testing of products. Such plans are developed taking into the consideration of relevant probability distributions of the lifetimes of the products under testing. In life testing, censoring schemes are adopted in order to save time and cost of life test. This paper attempts to determine reliability single sampling plans under hybrid censoring scheme assuming that the lifetime of the product follows exponentiated exponential distribution. Plan parameters are obtained corresponding to two specified points on the operating characteristic curve. The sampling plans protect the interests of both producer as well as a consumer. Selection of sampling plans is illustrated through numerical example.

1. Introduction

Reliability sampling plan is a statistical tool used in manufacturing industries for making decision about the disposition of lots based on the information obtained from a life test. Lifetime is a quality characteristic for some products. Sampling inspection for such products is carried out by conducting suitable life test. An acceptance sampling plan under which sampling inspection is performed by conducting life test upon the sampled products may be termed as reliability sampling plan. While compared to ordinary sampling plans, the execution of reliability sampling plans requires higher amount of sampling cost and inspection time. Censoring schemes are generally employed during the life test to make the inspection as a cost effective one. Time censoring (Type-I), Product censoring (Type-II) and hybrid censoring are some of the censoring schemes employed in the life test. Designing of reliability sampling plans considering censoring scheme is a suitable one when the inspection time is limited and available cost for inspection is less.

Reliability sampling plan is developed based on the probability distribution of the lifetime of the product. Many works can be found in the literature on designing of reliability sampling plans for various lifetime distributions. Epstein (1954), Balasooriya and Saw (1998) developed reliability sampling plans by attributes under hybrid and progressive censoring schemes respectively, assuming that the lifetime distribution is exponential. Kantam et al., (2001) and Rosaiah et al.,

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(2006) designed reliability sampling plans by attributes under a hybrid censoring scheme assuming that the lifetime distribution is log-logistic and exponentiated log-logistic distributions respectively. Aslam and Shahbaz (2007), Aslam et al., (2010) and Srinivasa Rao (2011) determined sampling plans based on generalized exponential distribution. Sriramachandran and Palanivel (2014) determined sampling plans based on exponentiated inverse Rayleigh distribution. Some special probability distributions like Birnbaum-Saunders distribution, Marshall-Olkin extended exponential distribution and Maxwell distribution were also considered for constructing sampling plans by Aslam and Kantam (2008), Srinivasa Rao et al., (2009) and Lu (2011). It should be noted that all these works were carried out assuming that the life testing is done under hybrid censoring scheme. Also, these works considering the consumer's risk only and ignoring the risk for the producer in rejecting the lots of good products.

Exponentiated exponential distribution, a most attractive generalization of the exponential distribution, introduced by Gupta and Kundu (1999) has received widespread attention. It is observed that the exponentiated exponential distribution has been used quite effectively to analysis lifetime data. In many cases, it is observed that it provides a better fit than the Weibull, gamma, log - normal or generalized Rayleigh distributions.

This paper attempts to determine reliability sampling plans based on exponentiated exponential distribution under hybrid censoring scheme corresponding to the specified levels of producer's risk and consumer's risk. Operating characteristic (OC) function of reliability single sampling plan (RSSP) under the conditions of exponentiated exponential distribution is derived in Section 2. The procedure of determining and operating the sampling plans is described in Section 3. In Section 4, construction of tables providing optimum sampling plans is discussed for some cases. Selection of sampling plan is illustrated with example. Results are summarized in Section 5.

2. Operating Characteristic Function of RSSPs under the conditions of Exponentiated Exponential distribution

A reliability single sampling plan is a procedure which can be used while taking decision about submitted lots by conducting a suitable life test to the randomly selected items from the lot. It can be defined by a set of four parameters (N, n, c, t), Where N is the lot size, n is the sample size, c is the acceptance number and t is the test termination time. The sampling plan can be implemented as follows:

(1) Select a set of n products randomly from the submitted lot of size N.
(2) Conduct the life test to the selected items considering t as the test terminated time. Observe the number of failed items X=x.
(3) Terminate the life test, if either at time t or $x > c$ before reaching time t, whichever is earlier.
(4) Accept the lot, if $x \leq c$ at time t; reject the lot if $x > c$ either at time t or earlier.

Let T be the lifetime of the product, which is distributed according to an exponentiated exponential distribution having the probability density function (PDF)

$$f(t;\gamma,\theta) = \frac{\gamma}{\theta}(e^{-\frac{t}{\theta}})(1-e^{-\frac{t}{\theta}})^{\gamma-1}, \qquad t>0, \gamma>0, \theta>0 \tag{2.1}$$

Here γ and θ are the shape and scale parameters respectively. The Exponentiated exponential distribution having the above PDF will be denoted by EE (γ,θ). As discussed in Gupta and Kundu (1999), the cumulative distribution function of EE (γ,θ) is given by

$$F(t;\gamma,\theta) = (1-e^{-\frac{t}{\theta}})^{\gamma}, \qquad t>0, \gamma>0, \theta>0 \tag{2.2}$$

With mean and variance respectively,

$$\mu = E(T) = \theta[\psi(\gamma+1) - \psi(1)] \tag{2.3}$$

and

$$\sigma^2 = V(T) = \theta^2[\psi'(1) - \psi'(\gamma+1)] \tag{2.4}$$

Here $\psi(\cdot)$ and $\psi'(\cdot)$ are the digamma and polygamma functions respectively, i.e.,

$$\psi(u) = \frac{d}{du}\Gamma(u), \qquad \psi'(u) = \frac{d}{du}\psi(u)$$

Where $\Gamma(u) = \int_0^\infty x^{u-1}e^{-x}dx$

The lot fraction nonconforming, p, can be calculated corresponding to each value of t/μ from

$$F_T(t/\mu) = p \tag{2.5}$$

The performance of a sampling plan may be analyzed using their OC functions. The OC function of a sampling plan is given by

$$P_a(p) = P(X \le c) = \sum_{x=0}^{c} P(X=x)$$

The probability distribution of X can be assumed appropriately as hypergeometric distribution. Schilling and Neubauer (2009) pointed out that, when $n/N \le 0.10$, n is large and p is small such that $np < 5$, the sampling distribution of X can be approximated by the Poisson (np) distribution. Under these circumstances, here, it is proposed that

$$P_a(p) = \sum_{x=0}^{c} \frac{e^{-np}(np)^x}{x!}$$

The value of lot acceptance probability$P_a(p)$ calculated using Poisson probabilities when c = 2, t = 500 hours, γ = 2 and n = 41, 65, 94 and 129. Values of p are computed for each set of (n,c,t,γ) using (2.5) for different values of μ. The OC curves of the sampling plans are presented in Figure 1. The figure shows that the value of $P_a(p)$ decreases to each combination of (n,c,t,γ), as p increases. It is a desirable property of a sampling plan safeguarding the interests of producer against rejecting the lots of good quality and the interest of the consumer against accepting the lots of poor quality. It can also be observed from the figure that the probability of a lot decreases when the sample size increases.

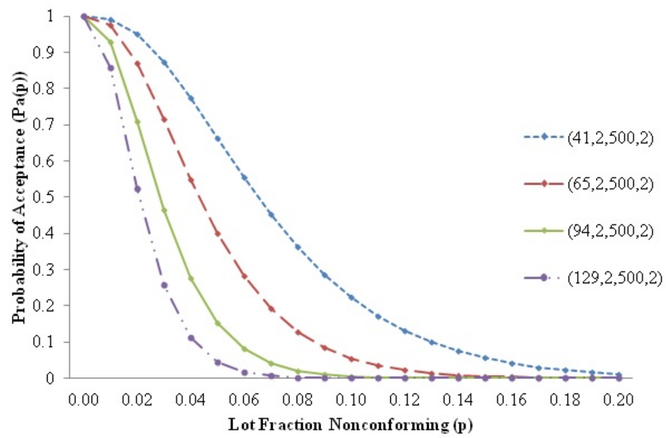

FIGURE 1. OC curve based on Poisson probabilities (c=2, t=500 hrs, $\gamma = 2$)

3. Determination of Plan Parameters under the conditions of Exponentiated Exponential Distribution

The optimum reliability single sampling plans are determined under the conditions of EE (γ, θ) distribution using the OC function defined by the Poisson probability distribution. A sampling plan is usually determined in such way that it protects simultaneously the producer and consumer. The protection to the producer and the consumer is ensured by specifying two points, namely, $(p_1, 1-\alpha)$ and (p_2, β) on the OC curve. Here, p_1 represents the acceptable quality level, α denotes the producer's risk, p_2 represents the limiting quality level and β denotes the consumer's risk. An optimum RSSP can be determined corresponding to the points satisfying the following conditions

$$P_a(p_1) \geq 1 - \alpha$$

and

$$P_a(p_2) \leq \beta$$

These conditions may be written as

$$\sum_{x=0}^{c} \frac{e^{-np_1}(np_1)^x}{x!} \geq 1 - \alpha \tag{3.1}$$

and

$$\sum_{x=0}^{c} \frac{e^{-np_2}(np_2)^x}{x!} \leq \beta \tag{3.2}$$

Different methods may be followed to determine the optimum values of n and c subject to (3.1) and (3.2). The iterative procedure described below is followed for finding the plan parameters. Thus, for given γ, t, μ_1, μ_2, α and β, optimum values of the plan parameters n and c can be determined as follows:

(1) For specified values of μ_1 and μ_2 with $\mu_1 > \mu_2$, calculate $\theta_1 = \frac{\mu_1}{[\psi(\gamma+1)-\psi(1)]}$ and $\theta_2 = \frac{\mu_2}{[\psi(\gamma+1)-\psi(1)]}$
(2) Corresponding to t, θ_1 and θ_2, determine $p_1 = F_T(t/\mu_1)$ and $p_2 = F_T(t/\mu_2)$
(3) Set c = 0
(4) Find the largest n, say n_L, such that $P_a(p_1) \geq 1 - \alpha$
(5) Find the smallest n, say n_S, such that $P_a(p_2) \leq \beta$
(6) If $n_S \leq n_L$, then the optimum plan is (n_S, c); otherwise increase c by 1
(7) Repeat Steps 4 through 6 until optimum values of n and c are obtained.

After determining n and c, the sampling inspection may be carried out for a submitted lot under hybrid censoring scheme as discussed in section 2.

4. Construcion of Tables

Values of n and c of the optimum reliability sampling plans are determined using Poisson probabilities for the some combination of $\gamma, t, \mu_1, \mu_2, \alpha$ and β. The producer's risk and consumer's risk are considered at two different levels such as $\alpha = 0.025, 0.05$ and $\beta = 0.05, 0.10$. Various levels of mean lifetime of the products as expected by the producer are taken as μ_1= 5000, 6000, 7000, 8000, 9000 and 10,000 hours respectively. Two different level of test termination time and one value for shape parameter γ are assumed as t = 500 and 750 hours and γ = 2 respectively. Various levels of mean lifetime of the product as expected by the consumer are taken as μ_2 = 1000, 1500, 2000, 2500, 3000, 3500 and 4000 hours respectively. Values of n and c of the optimum reliability sampling plans are presented in Table 1 through Table 4. In all tables, each cell entry (n, c) represents the optimum value of the pair (n, c) corresponding to the specified values of $\gamma, t, \mu_1, \mu_2, \alpha$ and β. Selection of plans from these tables for given requirements described in the following illustration.

Illustration

Let the lifetime of the products submitted for inspection be distributed according to the EE $(2, \theta)$. The mean lifetime of the products meeting the expectation of the producer and consumer are respectively μ_1 = 6000 hours and μ_2 = 2000 hours. Suppose that the quality inspector prescribes to censor the life test at t = 500 hours. Then, the values of acceptable quality level and limiting quality level can be computed as $p_1 = 0.01381$ and $p_2 = 0.09779$. If the producer's risk and the consumer's risk are prescribed as $\alpha = 0.025$ and $\beta = 0.05$, then the plan parameters may be obtained using Poisson probabilities from Table 1 as n = 94 and c = 4.

Now, the life test based lot-by-lot sampling inspection can be carried out as follows: A sample of 94 products may be selected randomly from the submitted lot. Life test may be conducted to all the sampled products. If the number of failures is 4 or less at 500 hours, the life test may be terminated. The lot may be accepted. On the other hand, if the fifth failure occures before t = 500 hours, terminate the life test. The lot may be rejected.

Table 1. Parameters of RSSPs under the conditions of EE($\theta, \gamma = 2$) Distribution with $\alpha = 0.025$,$\gamma = 2$ and t = 500 hours.

t=500, γ=2		μ_1	5000	6000	7000	8000	9000	10000
		t/μ_1	0.1000	0.0833	0.0714	0.0625	0.0556	0.0500
μ_2	t/μ_2	p_2 \ p_1	0.01940	0.01381	0.01032	0.00801	0.00639	0.00522
1000	0.5000	0.2784	(23,2) (20,2)	(23,2) (14,1)	(18,1) (14,1)	(18,1) (14,1)	(18,1) (14,1)	(18,1) (14,1)
1500	0.3333	0.1548	(51,3) (44,3)	(41,2) (35,2)	(41,2) (35,2)	(41,2) (26,1)	(31,1) (26,1)	(31,1) (26,1)
2000	0.2500	0.0978	(108,5) (82,4)	(94,4) (69,3)	(80,3) (55,2)	(65,2) (55,2)	(65,2) (55,2)	(65,2) (40,1)
2500	0.2000	0.0672	(234,9) (176,7)	(157,5) (139,5)	(137,4) (100,3)	(116,3) (100,3)	(94,2) (80,2)	(94,2) (80,2)
3000	0.1667	0.0489	(497,16) (388,13)	(296,8) (214,7)	(243,6) (190,5)	(188,4) (164,4)	(159,3) (137,3)	(159,3) (109,2)
3500	0.1429	0.0372	(1125,31) (910,26)	(589,14) (447,11)	(388,8) (317,7)	(319,6) (250,5)	(247,4) (215,4)	(247,4) (180,3)
4000	0.1250	0.0292	(3222,78) (2584,64)	(1115,24) (926,20)	(666,12) (528,10)	(494,8) (403,7)	(406,6) (318,5)	(360,5) (274,4)

In each cell, the first pair is the value of (n, c) corresponding to ($\alpha = 0.025$, $\beta = 0.05$) *and the second pair corresponding to* ($\alpha = 0.025, \beta = 0.10$).

Table 2. Parameters of RSSPs under the conditions of EE($\theta, \gamma = 2$) Distribution with $\alpha = 0.05$,$\gamma = 2$ and t = 500 hours.

t=500, γ=2		μ_1	5000	6000	7000	8000	9000	10000
		t/μ_1	0.1000	0.0833	0.0714	0.0625	0.0556	0.0500
μ_2	t/μ_2	p_2 \ p_1	0.01940	0.01381	0.01032	0.00801	0.00639	0.00522
1000	0.5000	0.2784	(18,1) (14,1)	(18,1) (14,1)	(18,1) (14,1)	(18,1) (14,1)	(18,1) (14,1)	(18,1) (9,0)
1500	0.3333	0.1548	(41,2) (35,2)	(41,2) (35,2)	(31,1) (26,1)	(31,1) (26,1)	(31,1) (26,1)	(31,1) (26,1)
2000	0.2500	0.0978	(94,4) (69,3)	(80,3) (55,2)	(65,2) (55,2)	(65,2) (40,1)	(49,1) (40,1)	(49,1) (40,1)
2500	0.2000	0.0672	(196,7) (157,6)	(137,4) (119,4)	(116,3) (100,3)	(94,2) (80,2)	(94,2) (80,2)	(94,2) (58,1)
3000	0.1667	0.0489	(423,13) (315,10)	(269,7) (216,6)	(188,4) (164,4)	(159,3) (137,3)	(159,3) (109,2)	(129,2) (109,2)
3500	0.1429	0.0372	(970,26) (758,21)	(490,11) (382,9)	(354,7) (250,5)	(283,5) (215,4)	(209,3) (180,3)	(209,3) (144,2)
4000	0.1250	0.0292	(2696,64) (2139,52)	(954,19) (769,16)	(581,10) (445,8)	(406,6) (318,5)	(360,5) (274,4)	(314,4) (229,3)

In each cell, the first pair is the value of (n, c) corresponding to ($\alpha = 0.05$, $\beta = 0.05$) *and the second pair corresponding to* ($\alpha = 0.05, \beta = 0.10$).

Table 3. Parameters of RSSPs under the conditions of EE($\theta, \gamma = 2$) Distribution with $\alpha = 0.025$,$\gamma = 2$ and t = 750 hours.

t=750, γ=2		μ_1	5000	6000	7000	8000	9000	10000
		t/μ_1	0.1500	0.1250	0.1071	0.0938	0.0833	0.0750
μ_2	t/μ_2	p_2 \ p_1	0.04059	0.02923	0.02204	0.01721	0.01381	0.01132
1000	0.7500	0.4561	(14,2) (12,2)	(14,2) (12,2)	(14,2) (9,1)	(11,1) (9,1)	(11,1) (9,1)	(11,1) (9,1)
1500	0.5000	0.2784	(33,4) (24,3)	(28,3) (20,2)	(23,2) (20,2)	(23,2) (14,1)	(23,2) (14,1)	(18,1) (14,1)
2000	0.3750	0.1851	(64,6) (51,5)	(50,4) (37,3)	(42,3) (37,3)	(35,2) (29,2)	(35,2) (29,2)	(35,2) (29,2)
2500	0.3000	0.1313	(130,10) (99,8)	(91,6) (71,5)	(70,4) (61,4)	(60,3) (51,3)	(60,3) (41,2)	(48,2) (41,2)
3000	0.2500	0.0978	(261,17) (206,14)	(161,9) (133,8)	(122,6) (95,5)	(94,4) (82,4)	(94,4) (69,3)	(80,3) (69,2)
3500	0.2143	0.0756	(600,34) (478,28)	(306,15) (236,12)	(208,9) (156,7)	(157,6) (123,5)	(140,5) (106,4)	(122,4) (89,3)
4000	0.1875	0.0601	(1694,85) (1365,70)	(581,25) (469,21)	(344,13) (277,11)	(262,9) (196,7)	(198,6) (155,5)	(175,5) (133,4)

In each cell, the first pair is the value of (n, c) corresponding to ($\alpha = 0.025$, $\beta = 0.05$) *and the second pair corresponding to* ($\alpha = 0.025, \beta = 0.10$).

Table 4. Parameters of RSSPs under the conditions of EE($\theta, \gamma = 2$) Distribution with $\alpha = 0.05$,$\gamma = 2$ and t = 750 hours.

t=750, γ=2		μ_1	5000	6000	7000	8000	9000	10000
		t/μ_1	0.1500	0.1250	0.1071	0.0938	0.0833	0.0750
μ_2	t/μ_2	p_2 \ p_1	0.0406	0.0292	0.0220	0.0172	0.0138	0.0113
1000	0.7500	0.4561	(14,2) (12,2)	(11,1) (9,1)	(11,1) (9,1)	(11,1) (9,1)	(11,1) (9,1)	(11,1) (9,1)
1500	0.5000	0.2784	(28,3) (20,2)	(23,2) (20,2)	(23,2) (14,1)	(18,1) (14,1)	(18,1) (14,1)	(18,1) (14,1)
2000	0.3750	0.1851	(57,5) (44,4)	(42,3) (37,3)	(35,2) (29,2)	(35,2) (29,2)	(35,2) (22,1)	(26,1) (22,1)
2500	0.3000	0.1313	(110,8) (90,7)	(81,5) (61,4)	(60,3) (51,3)	(60,3) (41,2)	(48,2) (41,2)	(48,2) (30,1)
3000	0.2500	0.0978	(224,14) (170,11)	(135,7) (108,6)	(108,5) (82,4)	(94,4) (69,3)	(80,3) (55,2)	(65,2) (55,2)
3500	0.2143	0.0756	(509,28) (404,23)	(258,12) (204,10)	(175,7) (140,6)	(140,5) (106,4)	(122,4) (89,3)	(103,3) (71,2)
4000	0.1875	0.0601	(1421,70) (1131,57)	(504,21) (393,17)	(303,11) (237,9)	(219,7) (176,6)	(175,5) (133,4)	(153,4) (1126,3)

In each cell, the first pair is the value of (n, c) corresponding to ($\alpha = 0.05$, $\beta = 0.05$) *and the second pair corresponding to* ($\alpha = 0.05, \beta = 0.10$).

5. Conclusion

Reliability sampling plans are determined in this paper for conducting life test based sampling inspection under hybrid censoring scheme when the lifetime of the product is followed exponentiated exponential distribution. The designed plan parameters will safeguard the interest of producer and consumer. Since the hybrid censoring scheme is employed to carry out life test, the implementation of sampling plans requires the lesser amount of time. Tables providing the optimum plans are presented for some specified strength.

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